OE FORM 6000, 2/69

DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
OFFICE OF EDUCATION

ERIC REPORT RESUME

ERIC ACC. NO.			
ED 030 016			
CH ACC. NO.	P.A.	PUBL. DATE	ISSUE
AA 000 376	24	30JUN69	RIENOV69

IS DOCUMENT COPYRIGHTED? YES ☐ NO ☐
ERIC REPRODUCTION RELEASE? YES ☐ NO ☐
LEVEL OF AVAILABILITY I ☐ II ☐ III ☐

AUTHOR
Suydam, Marilyn N.; Riedesel, C. Alan

TITLE
Interpretive Study of Research and Development in Elementary School Mathematics. Volume 1: Introduction and Summary What Research Says.

SOURCE CODE	INSTITUTION (SOURCE)
SYN71840	Pennsylvania State Univ., University Park
SP. AG. CODE	SPONSORING AGENCY
RMQ66004	Office of Education (DHEW), Washington, D.C. Bureau of Research

EDRS PRICE	CONTRACT NO.	GRANT NO.
1.00;12.85		OEG-0-9-480586-1352(010)

REPORT NO.	BUREAU NO.
	BR-8-0586

AVAILABILITY

JOURNAL CITATION

DESCRIPTIVE NOTE
255p.

DESCRIPTORS
*Elementary School Mathematics; *Mathematics Instruction; *Research Projects; Educational Research; *Development; Curriculum; *Research Utilization

IDENTIFIERS

ABSTRACT
The study involved the collection of (1) research reports, published through 1968, including journal articles and dissertation abstracts, and (2) information on developmental projects, supplemented by ERIC documents on funded research and Title III project reports. The journal articles were analyzed, categorized on ten aspects, evaluated, and summarized. Two instruments developed and tested for this purpose were used to evaluate; reports of tests of reliability on both instruments are included in this report. Dissertation abstracts, ERIC documents, and other project reports are summarized. Information derived from on-site visits to ten major projects is briefly summarized, and is followed by a taped interview with each project director. The research information about elementary school mathematics for 1900 through 1968 is synthesized, using the most valid of the findings of research to answer specific questions collected from editors of elementary school mathematics textbooks and college teachers of mathematics education courses. A list of the most applicable findings of research is also included. In a summary chapter, key research and developmental trends are discussed. (Authors)

GPO 870-390

BR 8-0586
PA 24
OE/BR

ED030 016

FINAL REPORT
Project No. 8-0586
Grant No. OEG-0-9-480586-1352(010)

INTERPRETIVE STUDY OF RESEARCH AND DEVELOPMENT IN ELEMENTARY SCHOOL MATHEMATICS

VOLUME 1:
INTRODUCTION AND SUMMARY
WHAT RESEARCH SAYS

Marilyn N. Suydam
C. Alan Riedesel

The Pennsylvania State University
University Park, Pennsylvania 16802

June 30, 1969

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

AA000376

FINAL REPORT
Project No. 8-0586
Grant No. OEG-0-9-480586-1352(010)

INTERPRETIVE STUDY OF RESEARCH AND DEVELOPMENT IN ELEMENTARY SCHOOL MATHEMATICS

VOLUME 1:
INTRODUCTION AND SUMMARY
WHAT RESEARCH SAYS

Marilyn N. Suydam
C. Alan Riedesel

The Pennsylvania State University
University Park, Pennsylvania 16802

June 30, 1969

The research reported herein was performed pursuant to a grant with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

ACKNOWLEDGMENTS

The directors of this study are very grateful to the graduate students who assisted in the vast amount of work involved. Charlotte Farris gave critical assistance at all stages, and, with John Howell and Richard Kohr, was responsible for much of the analysis of the articles. Judith Bechtel, Susan Farnum, Maria Martinez, and Evangeline Negron searched journals, summarized dissertation and report documents, and proofread countless pages.

Thanks are also due Beverly Brooks for typing this report, and to Joyce Axtell and Donna Ford for multilithing and collating.

To all those who gave assistance, including the directors of the developmental projects and Richard Elmendorf at U.S.O.E., thank you.

OVERVIEW

This Final Report of Phase I of the Interpretive Study of Research and Development in Elementary School Mathematics is bound in three volumes. Volume 1 describes the study and presents the summarized findings, in a form which should prove useful to teachers and principals. Volume 2, containing the compilation of categorized research reports, will possibly prove to be primarily of use to researchers. In Volume 3, reports of developmental projects are summarized; those teaching mathematics education courses may find these particularly helpful.

TABLE OF CONTENTS

VOLUME 1

Page

ACKNOWLEDGMENTS ii

OVERVIEW iii

PART I. INTRODUCTION AND SUMMARY 1

Rationale 1

Description of the Study 2

Procedures 3

Reactions from Target Audiences 4

Summary of Data 10

Key Trends 29

PART II. WHAT RESEARCH SAYS 33

Synthesis: Answers from Research 33

List of Applicable, Generalized Findings 174

APPENDIX A: Bibliography of Research Reports on Elementary School Mathematics, 1900-1968 177

VOLUME 1

PART I: INTRODUCTION AND SUMMARY

Rationale

Since the mid-1950's, a curriculum reform movement has brought many changes to the scope, the sequence, and the teaching of elementary school mathematics. Developmental projects large and small, aimed at innovation and diffusion, provided the initial impetus and the on-going thrust. Paralleling this attention to the formulation of new curricular materials was an increasing emphasis on research. The need to apply the findings of educational research to give direction to the teaching-learning process has intensified in recent years. Decisions about curriculum innovations must be related to knowledge about curriculum content and methods. A source of such knowledge and a foundation for decisions is research.

With the first realization at the beginning of the century that controlled experimentation might be a feasible technique for exploring many of the problems and issues which face educators, an overwhelming optimism took possession. The hopes of a panacea which would resolve all difficulties once and for all led to disillusionment, of course. Yet the concept remained that research can help to point the way toward certain decisions, even if many aspects of the educative process are not readily accessible to its tactics.

For the findings of educational research have not had the impact on curriculum decision-making in elementary school mathematics that they could have had. Included among the reasons for this unfulfilled need are:

(1) The findings have not always been readily available.

(2) The findings on many topics are equivocal or conflicting.

(3) Research reports are not always written in language which is clear to the non-researcher.

(4) The applicability of the results to a specific classroom or school situation is unclear.

This Interpretive Study of Research and Development in Elementary School Mathematics represents one phase of a plan to overcome these difficulties. It involves the synthesis, analysis and interpretation of the significant and valid findings of educational research and development projects in elementary school mathematics.

A primary task of an investigator is to determine the status of the topic to be explored, since research of the present and the future must be based on or indicate consideration of what has been done in the past. One of the difficulties which any researcher faces is locating those studies which will be of most use to him. For those interested in elementary school mathematics, this Study represents an extension of a previous U.S. Office of Education project, which was designed to provide such a compilation of the journal-published research for 1900 through 1965. While it is not yet completely comprehensive, the collection includes the vast majority of reports of research and developmental activities.

Description of the Study

The Study involved the collection of (1) research reports, published through 1968, including journal articles and dissertation abstracts, and (2) information on developmental projects, supplemented by ERIC documents on funded research and Title III project reports. The journal articles were analyzed, categorized on ten aspects, evaluated, and summarized. Two instruments developed and tested for this purpose were used to evaluate; reports of tests of reliability on both instruments are included in this report. Dissertation abstracts, ERIC documents, and other project reports are summarized. Information derived from on-site visits to ten major projects is briefly summarized, and is followed by a taped interview with each project director.

The research information about elementary school mathematics for 1900 through 1968 is synthesized, using the most valid of the findings of research to answer specific questions collected from editors of elementary school mathematics textbooks and college teachers of mathematics education courses. A list of the most applicable findings of research is also included. In a summary chapter, key research and developmental trends are discussed.

Procedures

The objectives for the Study all related to the field of elementary school mathematics, kindergarten through grade eight (excluding formal algebra). This includes all of the elementary school and a portion of the junior high, or, in current designation, the primary school and the middle school. The procedures which were followed in the Study were aimed at meeting these objectives:

1. To compile a list and collection of:
   a. Reports of research printed in journals published in the United States from 1966 through 1968; those for 1900-1965 are already available in our files.
   b. Dissertations completed in the United States through 1968.
   c. Reports of developmental projects and funded research.
2. To develop instruments to evaluate research reports, and to develop a questionnaire guide form for developmental projects.
3. To analyze and evaluate research reports from journal and non-journal sources and dissertations. Each study was categorized by mathematical topic and type of study, and, whenever appropriate, by design paradigm; statistical procedure, variables controlled, sampling procedure and size, type of test, grade level and duration will be noted. Major conclusions which appear consistent with the data of each study were noted. All of this information provides a basis for the syntheses.
4. To list and survey developmental projects, collecting information from on-site visitations and discussions with project and staff.
5. To synthesize the results of research, emphasizing findings from 1950-1968, since the development of "modern mathematics," but including significant, valid findings from before 1950.
6. To prepare a dissemination report for Phase II.

Reactions from Target Audiences

Three major target groups were identified for this study: (1) college teachers of elementary mathematics curriculum courses, (2) editors of materials dealing with mathematics for children in the elementary school, and (3) principals of elementary schools. These groups were chosen because each is in a unique position to effect change in elementary school mathematics instruction and content.

To ascertain the needs of these target audiences and to ensure the applicability and appropriateness of this study, a questionnaire was sent to all mathematics editors of elementary school textbooks and to a representative sampling of college teachers of elementary school mathematics. The third target audience was not surveyed because of (1) the probability that their perception of the needs of teachers would be highly correlated with those of the first two groups, and (2) the difficulty of obtaining a representative sampling. Essentially, the questionnaire consisted of four questions:

(1) Do you have a need to have readily translatable research information? Why or why not?

(2) In what form would research information best meet your needs?

(3) What types of research information do you need?

(4) Other comments and reactions.

Following are summaries of the responses obtained from the two groups. The reactions were used to determine the format of this report. In Part III, the majority of the specific questions which were noted by the instructors and editors are answered, with reference to specific research studies. Some questions, however, are not answered - or answerable - by research.

Summary of the findings of a questionnaire sent to instructors of courses on teaching elementary school mathematics. These instructors teach both pre-service and in-service courses for elementary school teachers. It should be noted that a number of professors teaching in small and medium-sized colleges indicated that mathematics was only one of several curricular areas of responsibility.

The first question on the questionnaire was, "Do you have a need to have readily translatable research information? Why or why not?" The responses were entirely "Yes." However, three of the twenty-six professors suggested the qualifier that there was not enough research to warrant considering its use. The comments that follow were typical of the responses: "I teach mathematics methods, science methods, and social studies methods. It is impossible for me to keep up with research information in all fields. Thus, easily read syntheses would help me." "I do not possess the research skills to analyze the research articles that I read. I would like short, valid answers to specific questions--based on research." "I want to give my students suggestions based on sound research. Most of the materials (books, pamphlets, etc.) are based on opinion rather than research. I would like to be able to answer student questions on the basis of research information." "We often say 'research says' when we mean 'I think that.' This is probably due to the fact that there is no single research source that an instructor can use to guide his thinking toward research answers."

The second question, "In what form would research information best meet your needs?", required that the professors check one of five given answers. Several checked more than one answer, therefore there were 32 check marks which were distributed over the five answers in the following manner:

Answers to specific questions	43%
Synthesis by mathematical topic	12%
Summaries of each research report	25%
Lists of valid findings and conclusions	15%
Other: please specify	5%

Comments by the instructors indicated that they would like a source of research abstracts and an answering of specific questions in a manner that could be used by their students.

The third question, "What types of research information do you need?", generated a host of questions which varied greatly in their degree of specificity. Five areas of interest can be considered.

One area of concern was the field of teacher education. Questions such as, "What is the most effective way in which to conduct in-service courses?" and "How can pre-service teachers learn teaching techniques without first-hand experience?" were typical of questions in this category.

A second area of interest was the field of mathematics learning theory. Questions such as "How effective is discovery teaching?", "What is the best method of teaching mathematics for retention and transfer?", "What is the role of behavioral objectives in mathematics teaching?", and "What is the role of developmental material and practice material?" were typical of this category.

A third area of interest was the provision for individual differences. "How can we best provide for individual differences in mathematics instruction?", "How effective is individualized instruction in elementary school mathematics?", "How does classroom organization help in the provision for individual differences?" and "What is the role of remediation, enrichment, and acceleration in elementary school mathematics?" were asked.

A fourth area can be considered the area of trends and current developments. Such questions as "How effective is the new mathematics?" and "What is the role of mathematics laboratories?" were typical of this area.

A fifth area dealt with the scope and sequence of topics in the elementary mathematics curriculum. Questions such as "Should multiplication and division be introduced simultaneously?", "At what level of maturity should mathematical properties be taught?" and "What is the optimum grade placement of topics in elementary school mathematics?" were asked.

The fourth question asked for "Other comments and reactions." The comments of the professors were quite varied. Some of the more interesting comments follow: "Why hasn't research in mathematics education been more effective?" "Why can't we get better dissemination of information concerning research in mathematics education?" "Shouldn't there be some type of clearinghouse of research in elementary mathematics education?" "We need to provide better means of keeping professionals up-to-date on research findings." "Means should be developed to update teachers and administrators concerning current research findings in elementary school mathematics." "In this age of mass communications we don't seem to be doing a very good job of communicating research findings."

From these comments it appears that professionals need efficient and effective means of increasing communication dealing with research.

Summary of the findings of a questionnaire sent to editors of elementary school mathematics texts. The response to the first question, "Do you have a need to have readily translatable research information?", was almost entirely "Yes," with only three of the fifteen editors responding otherwise. The reasons given for the response "Yes" can be grouped into two categories. The first category relates to the editors' interest in research as a device for keeping abreast of current trends and developments in elementary school mathematics. A typical comment from one editor is: "The educational publisher is in the business of serving a market. Every effort is made to be sure that what is published reflects the mainstream of thinking which is always emerging from the professional arena. It is our belief that this mainstream of thinking is largely influenced by current research. In order to stay in that mainstream, we must remain at all times fully informed as to its direction and the forces which are shaping its direction." Related to this is the second category of responses which relates primarily to research as evidence upon which to base decisions about future publications. As another editor stated, "We very definitely need research information because of the vast number of decisions which must be made which have a direct bearing upon education." The reasons for other-than-yes responses were either not listed, or indicated misunderstanding of the question.

The second question, "In what form would research information best meet your needs?", required that the editors check, with no absolute limit, any of five given answers. Therefore, there were twenty-four check marks which were distributed over the five answers in the following manner:

Answers to specific questions	8%
Synthesis by mathematical topic	17%
Summaries of each research report	25%
Lists of valid findings and conclusions	38%
Other: please specify	12%

Everyone checking this last answer specified topical abstracts. In general the editors seemed to indicate that brief reports of the important findings were the most valuable and usable.

The third question, "What types of research information do you need?", produced a very complex and extensive list of specific questions, which with some generalization can be considered in three separate areas of concern.

The first area of concern can be thought of as general trends and current practices in elementary school mathematics instruction. The specific information requested deals with school conditions and methods for teaching mathematics to low achievers. Information on materials such as calculators and games, and non-technical approaches in methods, were requested. Inner-city teaching materials were also of concern, as this question from an editor indicates: "What are the teachers and administrators of the inner-city schools looking for in educational material?" The need for information on practical applications and current practices is reflected in such questions as, "What percentage of teachers have accepted and are actually teaching 'new math'?" General information about educational material criteria was also requested.

The second area of concern regards student placement and readiness material placement and textbook reading level. Such questions as this were asked: "What effect is the increased number of topics now finding their way (through the demands of educational specialists and mathematicians) into all elementary school mathematics programs having upon the youngster?" Research answers regarding behavioral and educational objectives seem needed: as one editor asked, "What are suitable behavioral objectives for mathematics students at varying grade levels?"

The third such area can be considered the area of psychology in education, dealing with information about learning theory in general as well as in elementary education. Several asked, "What is the need for mathematics laboratories?" Related research in other fields and the future directions of educational psychological research in general was requested. The concern about materials is again reflected in questions such as, "Does the use of manipulative materials, such as Cuisenaire rods, string diagrams, balances, and student-constructed models, significantly improve mathematical understanding?" Research answers about time elements in dealing with concrete and abstract concepts are needed. The effect of textbook typing-face and other publishing-related questions such as, "What effect does color have on learning with the very young child? With the push to furnish all elementary schools with bright four-color books in order for them to be pretty, are we really retarding learning rather than helping it?", were asked. One editor questions: "The School Mathematics Study Group, Greater Cleveland Mathematics Program, Madison Project, Illinois Projects, Maryland Projects, etc., all seem to have various ideas on presentation for understanding for the elementary school child. Has any real research taken place that should give us the direction on what really should be done . . . ?"

The fourth question requested, "Other comments and reactions." This space was used by the editors to enlarge upon their responses to the first question. In general they felt that educational research was both difficult and expansive. One editor states: "The inappropriateness of standardized evaluation devices, the inability to control the teacher variable, and the exorbitantly high cost of research design, monitoring, and data processing makes research that is conducted of less significance than what is generally needed." They also indicated that it is becoming increasingly difficult to keep abreast of the current research: "Since there is such a vast amount of research being conducted today, anything that can be done to help us and other educators keep abreast of the research would be of tremendous service . . . " . They agreed that having information about current research would help them make better decisions about which textbooks to publish, which trends to keep alive, and which procedures to disseminate. One editor sums the situation in this rather interesting way. "It is an interesting fact that many multi-million dollar decisions are made by publishers with nothing more than a vague feeling as to the potential market for the product. The unfortunate part is that some important trends in education die out because publishers are not aware of their growth and do not publish the materials needed to keep the trend going. For example, it was necessary for the federal government to invest millions of dollars in S.M.S.G. in order to get the books that were needed to get the math revolution going. Perhaps if information about the trends in educational research were more readily available, private enterprise would respond to the needs of innovative programs." Indeed, perhaps the entire educational community would respond likewise.

Summary of Data

The data which resulted from the categorization and evaluation of the journal-published research reports presented in Volume 2 are summarized on several tables. A total of 305 analyses are listed. However, it must be noted that this does not represent the precise number of research reports since: (1) the possibility exists that some reports were not found and included; (2) some reports included more than one experiment, each of which was analyzed separately; and (3) some reports were duplicates, reporting the same research in more than one journal.

Journals. These research reports were found in 47 journals. The journals and the number of articles published by each are presented in Table 1. Three journals published almost half (48%) of the reports. Eight journals published 71% of the reports; twelve journals, 82%. The

TABLE 1

FREQUENCY OF REPORTS BY JOURNAL SOURCE

American Educational Research Journal	1
American Journal of Psychology	1
American Mathematics Monthly	4
Arithmetic Teacher	89
AV Communications Review	1
British Elementary Mathematics Journal	1
British Journal of Educational Psychology	15
California Journal of Educational Research	3
Catholic Education Review	1
Child Development	9
Childhood Education	1
Clearing House	2
Columbia Studies in Education	2
Duke University Studies in Education	9
Educational Method (Journal of Educational Method)	1
Educational Research	1
Elementary School Journal (Elementary School Teacher)	17
Indiana University School of Education Bulletin	27
Journal of Applied Psychology	1
Journal of Educational Psychology	8
Journal of Educational Research	13
Journal of Exceptional Children	2
Journal of Experimental Child Psychology	2
Journal of Experimental Education	4
Journal of Experimental Psychology	2
Journal of Genetic Psychology (Pedagogical Seminary; Pedagogical Seminary and Journal of Genetic Psychology)	8
Journal of Psychology	1
Journal of Research Development in Education	1
Journal of Research in Science Teaching	1
Journal of School Psychology	1
Journal of Teacher Education	3
Mathematics Teacher	11
National Elementary Principal	1

TABLE 1 (continued)

Peabody Journal of Education	1
Pittsburgh Schools	1
Primary Mathematics	1
Psychology in the Schools	2
School and Community	1
School Review	1
School Science and Mathematics	15
Supplementary Educational Monographs	29
Teaching Arithmetic: British Elementary Mathematics Journal	2
University of Illinois Bulletin	2
University of Iowa Monographs in Education	2
University of Missouri Bulletin	1
Wisconsin Journal of Education	1
Yearbook of Department of Elementary School Principals	2
	305

remaining reports (18%) were published in 39 journals. It should be noted that The Arithmetic Teacher, which has published the most reports since 1900, began publication in 1954.

Years. About half (158) of the reports were published prior to 1965; with the 799 reports included in the previous compilation, a total of 957 reports were found for the 1900-1965 period. For 1966, 56 reports were located; for 1967, 48 reports; for 1968, 43 reports. Thus, between 1900 and 1968, 1,104 reports have been found.

Mathematical topic and type of study. Table 2 presents two types of information: the frequency of mathematical topic and the frequency by type of study. The number of reports of experimental research was 78, and 112 reports of surveys were found. The distribution of reports gives some indication of the concern for various topics, as well as depicting the fact that some topics lend themselves more readily to one type of research. For instance, teacher pre-service (t-1) and teacher in-service (t-2) are most readily ascertained through surveys, while descriptive research was most frequently used for textbooks (d-1).

Cross-referencing. For maximizing readability, cross-referencing was done. The frequency of the cross-referencing adds more depth, for in many cases the topic which was cited first was selected arbitrarily. The cross-reference thus could have been the primary one. Table 3 summarizes the data on cross-references, and presents the total number categorized under each topic. It must be remembered that this cross-referencing was intended not to show each topic which was considered in a report, but to serve as an aid to the user who is interested in finding pertinent, helpful studies on a particular topic. Thus even though references might be made to addition in a particular study, if it did not seem that a user interested in finding studies on addition would find particular help in this report, it was not cross-referenced to addition. The subjective judgment of the reviewers operated in assigning these cross-references.

The totals within each mathematical category shift somewhat as all references are counted. The grouping which contains the most citations

TABLE 2

FREQUENCY OF REPORTS BY MATHEMATICAL TOPIC AND TYPE OF STUDY

Type of Study	d	s	c	a	r	F	e	Total
Historical development and procedures (a-1)	1							1
Values of arithmetic (a-2)								0
Planning and organizing for teaching (a-3)		2		2		4	9	17
Attitude and climate (a-4)		2				1	1	4
Drill and practice (a-5a)				1		1	3	5
Problem solving (a-5b)		2	1		3		7	13
Estimation (a-5c)								0
Mental computation (a-5d)								0
Homework (a-5e)							1	1
Review (a-5f)								0
Checking (a-5g)								0
Writing and reading numerals (a-5h)								0
Foreign comparisons (a-6)	3					1		4
	4	6	1	3	3	7	21	45

TABLE 2 (continued)

Type of Study	d	s	c	a	r	F	e	Total
Pre-first-grade concepts (b-1)		1		1				2
Readiness (b-2)								0
Logical order (b-3)								0
Quantitative understanding (b-4)								0
Content to be included in grade (b-5)								0
Time allotment (b-6)		1					3	4
		2		1			3	6
Counting (c-1)		4	1		1	1	1	8
Number properties and relations (c-2)		2	1					3
Whole numbers (c-3)		1						1
Addition (c-3a)		2						2
Subtraction (c-3b)							1	1
Multiplication (c-3c)		5					1	6
Division (c-3d)								0
Fractions (c-4)		1						1
Addition (c-4a)								0

TABLE 2 (continued)

Type of Study	d	s	c	a	r	F	e	Total
Subtraction (c-4b)								0
Multiplication (c-4c)		1					1	2
Division (c-4d)				1				1
Decimals (c-5)								0
Percentage (c-6)								0
Ratio and proportion (c-7)								0
Measurement (c-8)	1	1					1	3
Negative numbers (c-9)								0
Algebra (c-10)								0
Geometry (c-11)						1	1	2
Sets (c-12)								0
Logic (c-13)							1	1
Our numeration system (c-14)								0
Other numeration systems (c-15)		1					1	2
Probability and statistics (c-16)								0
	1	18	2	1	1	2	8	33

TABLE 2 (continued)

Type of Study	d	s	c	a	r	F	e	Total
Textbooks (d-1)	11	1						12
Workbooks (d-2)								0
Manipulative devices (d-3)				2		1	2	5
Audio-visual devices (d-4)								0
Programmed instruction (d-5)							4	4
Readability and vocabulary (d-6)	2	3						5
Quantitative concepts in other subject areas (d-7)								0
	13	4	0	2	0	1	6	26
Diagnosis (e-1)	1	4	2				1	8
Remediation (e-2)		2		1	1		2	6
Enrichment (e-3)				2		1		3
Grouping procedures (e-4)		1				2	1	4
Physical, psychological, and/or social characteristics (e-5)					1	1	1	3
Sex differences (e-6)		1						1
Socio-economic differences (e-7)		1		1		1	1	4
	1	9	2	4	2	5	6	29

TABLE 2 (continued)

Type of Study	d	s	c	a	r	F	e	Total
Testing (f-1)	4	6			1		1	12
Achievement evaluation (f-2)		3			3	5	2	13
Relation to achievement (f-3)					4			4
Age (f-3a)						1		1
Intelligence (f-3b)						1		1
Effect of parental knowledge (f-4)								0
Effect of teacher background (f-5)				1				1
	4	9	0	1	8	7	3	32
Transfer (g-1)						1	4	5
Retention (g-2)								0
Generalization (g-3)				1				1
Organization (g-4)	1	1			1		1	4
Motivation (g-5)							3	3
Piagetian concepts (g-6)		5			1	3	14	23
Reinforcement (g-7)							2	2
	1	6	0	1	2	4	24	38

TABLE 2 (continued)

Type of Study	d	s	c	a	r	F	e	Total
Pre-service (t-1)	5	24		4	1	4	6	44
In-service (t-2)	3	24		3	1	7		38
Background (t-3)		10		1		2	1	14
	8	58	0	8	2	13	7	96
Totals	32	112	5	21	18	39	78	305

TABLE 3

FREQUENCY OF REPORTS BY CROSS-REFERENCES AND TOTALS

Type of Study	Cross	Original	Total
Historical development and procedures (a-1)		1	1
Values of arithmetic (a-2)	1		1
Planning and organizing for teaching (a-3)	9	17	26
Attitude and climate (a-4)	3	4	7
Drill and practice (a-5a)	7	5	12
Problem solving (a-5b)	6	13	19
Estimation (a-5c)			0
Mental computation (a-5d)			0
Homework (a-5e)		1	1
Review (a-5f)			0
Checking (a-5g)			0
Writing and reading numerals (a-5h)			0
Foreign comparisons (a-6)		4	4
	26	45	71
Pre-first-grade concepts (b-1)		2	2
Readiness (b-2)	2		2
Logical order (b-3)			0
Quantitative understanding (b-4)	5		5
Content to be included in grade (b-5)			0
Time allotment (b-6)	2	4	6
	9	6	15

TABLE 3 (continued)

Type of Study	Cross	Original	Total
Counting (c-1)	2	8	10
Number properties and relations (c-2)	2	3	5
Whole numbers (c-3)	5	1	6
Addition (c-3a)	6	2	8
Subtraction (c-3b)	4	1	5
Multiplication (c-3c)	5	6	11
Division (c-3d)	2		2
Fractions (c-4)		1	1
Addition (c-4a)	2		2
Subtraction (c-4b)	2		2
Multiplication (c-4c)	1	2	3
Division (c-4d)		1	1
Decimals (c-5)			0
Percentage (c-6)			0
Ratio and proportion (c-7)			0
Measurement (c-8)	4	3	7
Negative numbers (c-9)	1		1
Algebra (c-10)			0
Geometry (c-11)	2	2	4
Sets (c-12)			0
Logic (c-13)	2	1	3
Our numeration system (c-14)			0
Other numeration systems (c-15)	2	2	4
Probability and statistics (c-16)			0
	42	33	75
Textbooks (d-1)	3	12	15
Workbooks (d-2)			0
Manipulative devices (d-3)	2	5	7

TABLE 3 (continued)

Type of Study	Cross	Original	Total
Audio-visual devices (d-4)	2		2
Programmed instruction (d-5)	5	4	9
Readability and vocabulary (d-6)	2	5	7
Quantitative concepts in other subject areas (d-7)	1		1
	15	26	41
Diagnosis (e-1)	3	8	11
Remediation (e-2)	2	6	8
Enrichment (e-3)	3	3	6
Grouping procedures (e-4)	3	4	7
Physical, psychological, and/or social characteristics (e-5)	3	3	6
Sex differences (e-6)	3	1	4
Socio-economic differences (e-7)	5	4	9
	22	29	51
Testing (f-1)	4	12	16
Achievement evaluation (f-2)	11	13	24
Relation to achievement (f-3)	4	4	8
Age (f-3a)	4	1	5
Intelligence (f-3b)	6	1	7
Effect of parental knowledge (f-4)			0
Effect of teacher background (f-5)	1	1	2
	30	32	62
Transfer (g-1)	6	5	11
Retention (g-2)	6		6
Generalization (g-3)	2	1	3

TABLE 3 (continued)

Type of Study	Cross	Original	Total
Organization (g-4)	5	4	9
Motivation (g-5)	2	3	5
Piagetian concepts (g-6)	3	23	26
Reinforcement (g-7)	1	2	3
	25	38	63
Pre-service (t-1)	3	44	47
In-service (t-2)	4	38	42
Background (t-3)		14	14
	7	96	103

is that for teaching: pre-service, in-service, and background (t); this is a function of this being a new category, not previously included. This is followed by basic concepts and methods of teaching them (c), educational objectives and instructional procedures (a), studies relating to learning theory (g), evaluating progress (f), individual differences (e), materials (d), and topical placement (b). The single categories in which the largest number of reports (15 or more) were categorized are:

(1) t-1: pre-service (47)
(2) t-2: in-service (42)
(3) a-3: planning and organizing for teaching (26)
(4) g-6: Piagetian concepts (26)
(5) f-2: achievement evaluation (24)
(6) a-5b: problem solving (19)
(7) f-1: testing (16)
(8) d-1: textbooks (15)

Design paradigm: The frequency distribution for the design paradigms is presented on Table 4. Those more frequently noted were:

(1) 2.6: posttest only, control group, matched, n = students (7)
(2) 3.21: non-equivalent control group, pretest-posttest (7)
(3) 2.9: three or more groups, pretest-posttest, matched, n = classes (6)
(4) 2.16: three or more groups, posttest only, randomized, n = students (6)

Analysis of Table 5 reveals a problem which is shown in several ways: sampling and/or the way in which a researcher reported the sampling for his experiment was a point of great variability and ambiguity. The "3." categories in general indicate a question about sampling. These categories account for 40% of the total. Perception is quite obviously part of the problem, but nebulous writing needed to be clarified in at least this many cases. In some others, it can only be hoped that the reviewers actually correctly interpreted what the research did.

Table 5 shows that, while only 3% of the most recent studies (1966-1968) used no control groups, 45% of the research had questionable

TABLE 4

FREQUENCY OF REPORTS BY DESIGN PARADIGM

1.1	0
1.2	3
1.3	0
2.1	3
2.2	4
2.3	0
2.4	3
2.5	0
2.6	7
2.7	0
2.8	0
2.9	6
2.10	1
2.11	4
2.12	0
2.13	3
2.14	0
2.15	4
2.16	6
2.17	0
2.18	1
2.19	1
2.20	0
2.21	1

3.1	1
3.3	1
3.4	3
3.5	1
3.7	0
3.8	3
3.9	0
3.11	0
3.13	1
3.15	1
3.17	0
3.18	3
3.19	5
3.20	0
3.21	7
3.22	2
3.23	0
3.25	2
3.27	0
3.28	0
3.29	0
3.30	1

design techniques. Thus 48%, or almost one-half of the most recent research, did not meet the requirements of good research due to no control group, non-equivalent control groups, questionable sampling techniques, insufficient information, or the use of an incorrect n.

TABLE 5

FREQUENCY OF USE OF TYPES OF
DESIGN PARADIGM BY YEARS

Paradigm Type	1900-1929 n	1900-1929 %	1930-1950 n	1930-1950 %	1951-1965 n	1951-1965 %	1966-1968 n	1966-1968 %
1.	1	25	0	0	0	0	2	3
2.	3	75	4	100	6	67	31	52
3.	0	0	0	0	3	33	27	45

Statistical procedure. On Table 6 the statistical procedures which were noted are tabulated. It must be stated that this is not a completely accurate figure, for not all instances of statistical procedure were tabulated. Thus this table should be considered to indicate the most obvious trends in statistical uses. It is not an exact tabulation. Descriptive statistics are noted in almost 2/3 of the reports. The other techniques most noted were:

(1) 3.4: t-test (59)
(2) 3.2: analysis of variance (47)
(3) 6.4: correlation (31)
(4) 2.6: Chi square test for independence (25)
(5) 3.5: analysis of covariance (18)
(6) 3.3: F-test (17)

design techniques. Thus 48%, or almost one-half of the most recent research, did not meet the requirements of good research due to no control group, non-equivalent control groups, questionable sampling techniques, insufficient information, or the use of an incorrect n.

TABLE 5

FREQUENCY OF USE OF TYPES OF
DESIGN PARADIGM BY YEARS

Paradigm Type	1900-1929		1930-1950		1951-1965		1966-1968	
	n	%	n	%	n	%	n	%
1.	1	25	0	0	0	0	2	3
2.	3	75	4	100	6	67	31	52
3.	0	0	0	0	3	33	27	45

Statistical procedure. On Table 6 the statistical procedures which were noted are tabulated. It must be stated that this is not a completely accurate figure, for not all instances of statistical procedure were tabulated. Thus this table should be considered to indicate the most obvious trends in statistical uses. It is not an exact tabulation. Descriptive statistics are noted in almost 2/3 of the reports. The other techniques most noted were:

(1) 3.4: t-test (59)
(2) 3.2: analysis of variance (47)
(3) 6.4: correlation (31)
(4) 2.6: Chi square test for independence (25)
(5) 3.5: analysis of covariance (18)
(6) 3.3: F-test (17)

TABLE 6

FREQUENCY OF USE OF STATISTICAL PROCEDURES

1.1	84
1.2	0
1.3	28
1.4	134
1.5	17
1.6	98
1.7	5
1.8	10
1.9	3
1.10	1
1.11	0
1.12	1
1.13	2
	383
2.1	2
2.2	0
2.3	2
2.4	2
2.5	2
2.6	25
2.7	0
2.8	0
2.9	0
2.10	0
	33
3.1	0
3.2	47
3.3	17
3.4	59
3.5	18
3.6	2
3.7	1
3.8	0
3.9	0
3.10	0
3.11	0
3.12	0
3.13	2
3.14	0
3.15	6
3.16	0
3.17	0
3.18	0
3.19	0
	152
4.1	0
4.2	0
4.3	4
4.4	3
4.5	0
4.6	2
4.7	0
	9
5.1	0
5.2	4
5.3	0
	4
6.1	8
6.2	1
6.3	1
6.4	31
6.5	1
6.6	0
6.7	0
6.10	1
	43

Qualitative value. Analysis of the qualitative values which resulted from application of the instruments for evaluating research reports shows a range from 10 to 39 for experimental studies and 19 to 42 for surveys. The frequency for each is presented on Table 7.

TABLE 7

FREQUENCY OF REPORTS BY
QUALITATIVE VALUE

Value	Type of Study		Value	Type of Study	
	s	e		s	e
9	0	0	28	7	3
10	0	1	29	5	4
11	0	1	30	9	3
12	0	0	31	13	2
13	0	1	32	6	2
14	0	2	33	8	2
15	0	1	34	5	1
16	0	3	35	3	2
17	0	3	36	2	1
18	0	7	37	4	2
19	1	2	38	1	1
20	2	4	39	2	1
21	1	3	40	1	0
22	4	8	41	0	0
23	4	7	42	1	0
24	3	3	43	0	0
25	7	4	44	0	0
26	7	3	45	0	0
27	15	1			

Authors. Analysis revealed that 20 researchers wrote 3 or more articles; 14 of these wrote 4 or more; 11 wrote 5 or more. Table 8 indicates these authors and the number of reports. This includes all types of studies, not only experimental. It is of interest to note that frequently the researcher was primarily concerned with one topic. Brownell and Carper attempted to ascertain facts about whole number operations. Dutton was concerned mainly with the attitudes of prospective teachers. Smith and Eaton did an extensive survey of textbooks. The readability and vocabulary of arithmetic materials, and diagnostic studies of various types were the main topics of Buswell.

TABLE 8

FREQUENCY OF REPORTS BY AUTHOR

Ashlock; Flournoy; Houston; Lerch; Paige; Phillips	3
Carper; DeVault; John	4
Reys; Suppes; Welch	5
Judd; Woody	6
Terry	7
Smith	8
Eaton	9
Buswell	10
Dutton	11
Brownell	17

Key Trends

The primary trends discernible in research and development in elementary school mathematics reflect the growing and continuing involvement of larger numbers of people in these processes. Since 1957, the curriculum reform movement has included the establishment of a dozen major mathematics projects, plus innumerable smaller efforts--most of which are supported in whole or part by Federal or foundation monies. Not all of these projects have had the desired impact, and others are still too close to the initial evolvement stage. No one can deny the impact of S.M.S.G. on the curricular materials used in the elementary school. Ideas from the Madison Project and the University of Illinois Arithmetic Project have often filtered into textbooks, frequently without the source being identified. For instance, use of the number line was promoted to a great extent by its successful use in the Illinois materials. Newer projects, such as CSMP/CEMREL, are now incorporating ideas from earlier projects as well as testing new ones within a new structure: their potential is yet to be ascertained.

When the curriculum development projects are studied, it is evident that few of them involve true classical research components. The most successful or at least promising efforts, however, are those which include direct involvement with children as the program is being developed. As suggestions and ideas are proposed, they are tested with the "target audiences." There is a continual attempt to ascertain the "teachability" and "learnability" of the material.

This may provide a clue: maybe the most useful answers for the curriculum are evaluation answers rather than research answers, as Guba and others have suggested. The question asked by researchers is often, "Does program X teach better than program Y?" Maybe the far more appropriate question is, "Does program X teach effectively?" This can be carried to extremes, of course; there is still a need for basic research, which will contribute to the development of learning theory, as well as applied research which will answer specific questions and specific needs.

In long-term development projects, evaluation may be the most appropriate over-all procedure. For smaller components of the program, experimentation with specific aspects should be done. Thus there would be an attempt to get answers from research to put into the program to be evaluated. Not all of the questions which arise in connection with a project are answerable by research, since the total contextual and conceptual framework of the project must be considered. Some later actions may be dictated by earlier decisions. Some may be directed by the philosophical orientation. Some are derived by implication or controlled experimentation over a long period of time. Other choices, however, may be explicitly studied--and the results of such research may be applicable to many other classrooms.

As reports of research in elementary school mathematics are surveyed, one becomes increasingly aware of how the findings seem to be almost randomly distributed across the matrix of the mathematics curriculum. Why might this be so? First, not all of the questions about what and how to teach can be answered by research--many of them are philosophical. Second, most research is still conducted at the doctoral level, by one-shot researchers who frequently (and perhaps logically) are interested as much in how efficiently a study can be done as in how much it will contribute to the pattern of knowledge. However, there would be far less known about the teaching of elementary school mathematics were it not for doctoral dissertations. At the same time, there is a need for a greater proportion of the research to be focused on answering related questions.

There is a trend toward theory and model building, directed toward the development of a theory of instruction for teaching mathematics. A computer can accumulate and analyze a great deal of data, which are being used to provide empirical support for some of the theory-building. While little of this type of work appears in these volumes, it should help in the future in giving direction to the educational research process.

Another discernible trend is related to the amount of research being done. Research reports have been proliferating rapidly, especially since 1960. Two problems are still evident: (1) the time lag between the actual research work and the appearance of the published report, and (2) the failure to communicate all necessary information to the reader. Little improvement in quality can be noted, although this is in part a function of the reporting process as well as the quantity. Strategies to help educators gather, assimilate, and use the results of research are increasingly intensive and varied, ranging from computer-backed resource centers to interpretive study projects.

In analyzing the content-related findings of research, since the beginning of the curriculum reform movement in the late 1950's, it is of interest to note several trends:

(1) The reform movement emphasized mathematical structure. As a result, more abstract concepts and experiences were presented. During the past few years, however, there is again increasing emphasis on the use of objects and other manipulative materials to provide a foundation at various stages.

(2) Ten years ago, there was great emphasis on sets and on non-decimal numeration. Now both are stressed less, primarily because it has not been possible to demonstrate the relationship of each to increased understanding of our numeration system.

(3) One area which has received continued emphasis is geometry, with continued exploration of how much can be taught and effective teaching procedures.

(4) The importance of various teaching strategies and of affective learning is increasingly being recognized.

Undoubtedly many other trends will be observable to the reader as he explores the materials which follow.

PART II: WHAT RESEARCH SAYS

Synthesis: Answers from Research

This section contains the synthesis of answers from research. The questions supplied by target audiences are answered by reference to specific studies. Other questions to which research has indicated an answer are also included. Only findings evaluated as valid are cited, unless limitations indicate otherwise. The studies cited are included in Appendix A; those in which the year is prefaced by "DA" indicate that they are references from Dissertation Abstracts.

Answers from Research: Historical development and procedures (a-1)

What effect on the learner is produced by using historical procedures?

McPherson (DA 1968) found that use of historical materials facilitated comprehension of selected mathematical concepts. Few teachers developed historical comparisons or stressed underlying principles, as Bradley and Earp (1966) suggested should be done.

Are historical algorithms most appropriate for remedial work, enrichment, or the regular program?

This question has not been answered by research, though there is some indication that understanding of historical algorithms may be most appropriate for enrichment work.

What mathematics is used by pupils outside the classroom?

Ellsworth (1941) found that in an urban area children used telling time, money, counting, and reading numbers most frequently, while measuring area and operations with fractions were used least often. Moseley (1938) found the order of use at sixth grade was money, subtraction, addition, multiplication, measuring, division, and fractions, with games, shopping, and chores providing the greatest occasions for use. Smith (1924) found that first graders used arithmetic in stores, in games requiring counting, in reading Roman numerals on the clock and Arabic numerals on book pages. Addition and counting were most frequently used. Addition was also used most by third graders (Wahlstrom, 1936), and division very rarely used. Willey (1943) ordered the uses as money, measurement, time, objects, pets, and distance, finding counting, fractions, and subtraction were most often needed in problems.

Is the "new" mathematics superior to "old" mathematics?

The emphasis upon "new" or "modern" mathematics during the past ten years has caused parents and teachers alike to ask the question stated above. Clearly it is impossible to give a single definitive answer to the questions since there are many types and varieties of "modern mathematics." However, the research studies cited below have delved into some phases of evaluation of current programs in elementary school mathematics.

Ruddell (1962) studied four accelerated seventh-grade classes, two of which used commonly accepted traditional programs and two which used a program of modern orientation. He found that pupils taught in the modern program scored as high or higher (statistically significant) than similar children taught in a traditional program. Simmons (DA 1966) also found that students taught under a modern program scored higher. Payne (1965) surveyed the literature and found modern programs to be as effective as traditional programs in developing traditional mathematical skills and that there is evidence to support the conclusions that modern materials may be appropriate for a wide range of student abilities.

Hungerman (1967) compared ten classes at the sixth grade level who had studied the School Mathematics Study Group program during grades 4, 5, and 6 with ten classes who had studied a conventional arithmetic program during grades 4, 5, and 6. She found that (1) traditional achievement data (California Achievement Test) significantly favored the non-S.M.S.G. groups while contemporary achievement data (California Contemporary Mathematics Test) significantly favored the S.M.S.G. groups, (2) attitude toward mathematics was similarly positive in both groups, and (3) socio-economic level demonstrated little or no relationship to either achievement or attitude toward mathematics. Grafft (DA 1966) found that intermediate grade pupils taught by an S.M.S.G. program understood principles of multiplication better.

Several studies occurred involving junior high school students using S.M.S.G. materials. Friebel (1967) studied six classes of pupils randomly assigned to either S.M.S.G. or the state-adopted text Understanding Arithmetic 7 by McSwain and others. He found that the general achievement of the two groups was similar, but that the S.M.S.G. group achieved significantly superior growth in arithmetic reasoning and in concepts dealing with measurement. Cassel and Jerman (1963) studied achievement results from 262 students in grades 7, 8, and 9. This preliminary evaluation of S.M.S.G. instruction was based largely on a comparison of test scores for pupils enrolled in S.M.S.G. courses with corresponding scores for matched pupils in traditional courses. S.M.S.G. pupils had statistically significantly higher arithmetic and algebra test scores than the matched traditional pupils. Williams and Shuff (1963) studied 678 pupils in grades 7, 8, and 9 and compared S.M.S.G. pupils with pupils in traditional courses. They found (1) no significant differences at the 7th grade level, (2) significant differences at the 8th grade level favoring the traditional groups, and (3) no significant differences in the 9th grade groups. Osburn (DA 1966) reported no significant changes in skill development after use of S.M.S.G. materials.

Scott (1967) studied the summer loss of modern (Greater Cleveland Mathematics Program) and traditional elementary school mathematics programs. He found that while most children suffer some summer loss in arithmetic achievement, there appears to be no systematic relationship between the "modern" and "traditional" and students' retention of previously learned mathematical concepts.

How effective is an "activity" approach to teaching elementary school mathematics?

The current emphasis upon mathematics laboratories and the stress on correlation between science and mathematics will certainly generate research studies connected with these patterns. A somewhat similar movement occurred at an earlier point in time. The summary that follows deals with integrative activity programs. It should be noted that none of the studies described below would stand up to the present criteria for valid research.

A number of studies show results favoring an activity program. Collings (1933) found that pupils taught by an activity curriculum achieved higher scores on all arithmetic measures than pupils from a conventional subject curriculum. Harap (1934, 1936, 1937) presented findings that favored activity programs. Hopkins (1933) found that children taught in an experience curriculum achieved scores comparable to the norms established for pupils taught in a traditional curriculum. Other studies which produced results favoring activity curriculums were reported by Passehl (1949), Pistor (1934), Williams (1949), and Wrightstone (1935a, 1935b). Wilson reported evidence favoring an informal (activity) approach combined with a strong emphasis upon specific drill.

Some studies produced results unfavorable to the activity curriculum. Gates (1926) found that a systematic method resulted in higher achievement than the opportunistic method. Jersild (1939) found that groups in a non-activity program maintained a substantial advantage over those in the activity program both in arithmetic computation and arithmetic reasoning. Wrightstone (1944) found as part of an evaluation of six years of experimentation that the arithmetic scores of pupils in activity groups were significantly lower than those in the non-activity groups.

How effective is the "meaning" method?

Since the early 1930's, mathematics educators have advocated that "pupils should understand the mathematics they are taught." This goal gave rise to the "meaning approach" to teaching elementary school mathematics. Typically the meaning approach is contrasted with the rote learning or rule approach in which the pupil does not develop an understanding of the rationale of the mathematics he is taught. Certainly the meaning approach laid the foundation for "modern mathematics."

The majority of studies which involve the meaning approach are remarkably consistent in their findings. Typically researchers found that (1) rote rule and meaning produce about the same results when immediate computational ability is used as a criterion, (2) when retention is used as a criterion the meaning method is superior to the rote rule method, (3) greater transfer is facilitated by the meaning method, and (4) the meaning method produces greater understanding of mathematical principles and comprehension of complex analysis. (See: Brownell, 1949; Dawson, 1955; Greathouse, DA 1966; Krich, 1964; Miller, 1957; and Rappaport, 1958, 1963). Specific findings for use of this method can be found under the sections dealing with the mathematical topics taught in the elementary school.

What organizational patterns facilitate learning in elementary school mathematics?

Since the beginning of public education in the United States administrators and teachers have searched for the perfect organizational pattern. The research reported below continues this search.

Ungraded programs. A number of research studies have focused upon the use of non-graded or multi-graded patterns of instruction. Finley (1963) and Metfessel (1960) found no significant difference between multi-grade and single grade groups. Hart (1962) found that non-graded primary pupils achieved better in mathematics than graded groups. He dealt with only 100 pupils. In contrast Skapski (1960) found mathematics achievement to be higher in the primary graded groups. It seems safe to assume that achievement differences in mathematics are affected more by other variables than the variable of graded versus non-graded.

Team teaching. Jackson (1964) studied 14 classes in grades 5 and 6, some team teaching and some self-contained homeroom sections. He found there were no significant differences in achievement between the two groups. The findings of Lindgren (DA 1968) were similar. Sweet (1962) surveyed pupils and teachers and found varying opinions concerning the advantages and disadvantages of team teaching in grade seven. Piage (1967) tested 300 seventh and eighth grade pupils, some in team teaching and some in single teacher classes. He found that team teaching appeared to be more successful at eighth grade than seventh grade level. Neither grade level of pupils indicated team teaching to be the favorite form of instruction. Crandall (DA 1967) found that intermediate grade pupils achieved more in self-contained classrooms than those taught by team teaching.

Departmentalization. Periodically subject matter leaders suggest that departmentalization should be used so that the subject matter expertise of teachers can be brought into focus. Attempts to isolate the effect of departmentalization are fraught with difficulties. Thus, it is extremely difficult to conduct a valid study concerned with this topic. The findings described below should be considered in this light. Gibb and Matala (1961, 1962) studied 34 fifth and sixth grade classes in terms of comparing the use of special (departmentalized) teachers in science and mathematics. They found that (1) there were no significant differences in achievement between children taught in self-contained classrooms and those taught by special teachers, and (2) there was no evidence that special teachers increased pupil interest in mathematics. Gerberich and Prall (1931) found differences favoring departmentalization. It should be noted that they were dealing with a mathematics curriculum quite different from today's. Price (1967) statistically equated two fifth-grades and compared departmentalization and self-contained. He found no significant differences. The findings of Grooms (DA 1968) were similar, while Eaton (1944) reported that achievement in nondepartmentalized classes was higher.

Discussion. Many studies have been conducted concerning mathematics achievement and instructional grouping. Davis and Tracy (1963) present an excellent summary of the findings of the 1950's and early 1960's, finding that studies do not reveal any clear-cut advantages for special grouping procedures.

Study of the research conducted on administrative organizational programs to meet individual pupil needs is inconclusive. A proponent for one plan can find studies that verify his stand. Conversely, an opponent of the same program can find studies that show that this plan works no better than the typical administrative, single teacher, graded pattern. Perhaps the most important implication of the various studies is that good teachers are effective regardless of the nature of classroom organization.

How effective are "discovery type" of teaching approaches compared to "expository type" teaching approaches?

A number of good studies have been addressed to this question. An excellent study by Worthen (1968) with 432 pupils at the fifth and sixth grade level compared discovery and expository presentation. From his findings he suggests that (1) if pupil ability to retain mathematical concepts and to transfer the heuristics of problem solving are valued outcomes of education, discovery sequencing should be an integral part of the methodology used in presenting mathematics in the elementary classroom, and (2) if immediate recall is a valued outcome of education, expository sequencing should be continued as the typical instructional practice used in elementary classrooms. It is suggested that the Worthen study is well worth reading by all interested in discovery-type teaching.

Henderson and Rollins (1967) found three types of inductive (discovery) strategies to be effective in teaching concepts and generalizations. Armstrong (DA 1968) reported that the inductive mode fostered the learning of operations, while the deductive mode resulted in greater learning of mathematical properties. Meconi (1967) used programmed materials to compare rule and example, guided discovery, and discovery techniques at the eighth and ninth grade level. He found that pupils learned effectively with each technique. The findings of Hanson (DA 1967) were similar.

Scandura (1964a, 1964b, 1964c) conducted several related studies concerned with exposition versus discovery. He found that (1) discovery pupils were better able to handle problem tasks, (2) the discovery group took longer to reach the desired level of facility, and (3) exposition pupils generally used the algorithm taught while discovery subjects seemed more reliant.

Do elementary pupils like mathematics?

It is a widely accepted notion that mathematics is disliked by most pupils. However, results of numerous surveys contradict this notion. Many studies provide results that show pupils frequently select arithmetic as their favorite subject (Inskeep, 1965; Mosher, 1952; Rowland, 1963). Several other surveys report arithmetic as being above average as a preferred subject (Anderson, 1958; Chase, 1949; Curry, 1963; Herman, 1963; Stright, 1960; Greenblatt, 1962). Chase (1958), Curry (1963), and Dutton (1956) found middle grade boys rating their liking for arithmetic slightly higher than girls, but Stright (1960), when including lower grades, found girls showing slightly higher preference. Chase (1949) reported that New England pupils rated arithmetic slightly higher than pupils in the Southwest.

Do pupils show a preference for modern or traditional mathematics?

Generally, it has been found that pupils who like mathematics like either modern or traditional programs. Abrego (1966) compared pupil attitudes towards modern versus traditional mathematics and found that pupils who liked one type, liked the other. Hungerman (1967) found that pupils holding positive attitudes towards both conventional and contemporary mathematic programs. Dutton (1968) reported a slight increase in attitudes towards modern mathematics when compared with pupil attitudes of ten years before.

How does the attitude of the teacher affect the attitude of the pupil?

This question cannot be answered directly, but the relationship of the teacher and pupil attitudes has been investigated, with differing results. Inskeep (1965) found no relationship between teacher and pupil attitudes, but Chase (1958) found that the pupils of teachers who preferred arithmetic appeared to favor it themselves. With high intelligence pupils, Greenblatt (1962) reported that the preference of teachers corresponded strongly with that of the pupils.

How does the classroom climate affect pupil learning in mathematics?

The influence of differing classroom climates on arithmetic achievement has been investigated by Guggenheim (1961), who found no significant differences for classrooms that were and were not dominated by the teacher. The amount and kind of interaction was investigated by Hudgins and Loftis (1966), who found that teachers initiated interaction more frequently with average-ability pupils than with high-ability pupils.

What procedures improve pupil attitudes towards mathematics?

When arithmetic is taught as a skill that has practical value and is useful in out-of-class situations, attitudes become more positive. Studies by Dutton (1956), Lyda and Morse (1963), Malone and Freel (1954), and Stokes (1956) reached conclusions to support this statement. Stokes (1958) found higher sustained attention of pupils and Hunnicutt (1944) found activity methods associated with awareness of out-of-class use of arithmetic. Fedon (1958) found a positive increase in attitude when problem solving was related to experiments. Both presentation of arithmetic by television (Kaprelian, 1961) and specific review (Burns, 1956) seemed to create more positive attitudes. Dutton (1956) found pupils to report lack of understanding, difficulty, poor achievement, and boring aspects of arithmetic as major reasons for their dislike of arithmetic.

What is the relationship between achievement, ability, and attitude?

The contribution of attitude and interest to achievement is not easily measured because of other variables, but research by Bassham, Murphy and Murphy (1964), Dean (1950), Lyda (1963), Powell (1966), and others indicates there is a positive relationship. Anttonen (DA 1968) is among those who found no relationship. Greenblatt (1962) found girls with high arithmetic achievement had more positive attitudes. Rowland and Inskeep (1963) found a feeling of success increased preference and attitudes for arithmetic. The relationship of intelligence, which cannot be disassociated from achievement, was investigated by Rice (1963), Greenblatt (1962), and Stephens (1960), who found gifted or accelerated pupils had a higher interest in arithmetic. A study having related findings is one by Feldhusen and Klausmeier (1962), in which a significant relationship was found between high anxiety and low arithmetic achievement for low I.Q. pupils.

How much time should be devoted to drill and practice?

Hahn and Thorndike (1914) reported that longer periods of about 20 minutes were most effective, while Meddleton (1956) cited stronger evidence to show that systematic, short review work produces higher achievement. In a more recent well-done study, Shipp and Deer (1960) found that less than 50% of class time should be spent on practice activities, since increased achievement resulted when up to 75% of the time was spent on developmental activities. This finding has been supported by Shuster and Pigge (1965), Zahn (1966), and Hopkins (DA 1966).

What type of drill procedures are most effective?

Greene (1930) summarized studies which showed that drill must be constructed to fit a particular purpose and type of use, and this connection of drill with a purpose and the topic under study has been found to be of most help in more recent studies, too. Motivation and functional experiences are important (Harding and Bryant, 1944; Hoover, 1921; Lutes, 1926). Distributed practice is most helpful, rather than concentrated practice, according to Knight (1927). Children should use practice materials on their own difficulty level and progress at their own rate (Moench, 1962). Varying the type of drill and the use of "frames" were found to be effective by Sandefur (DA 1966).

Where in the sequence of learning mathematics is drill most effective?

After effective teaching is the time for drill, stated Brownell and Chazal (1935), and this has been generally supported and accepted.

The solving of verbal or word problems has long been one of the areas of elementary school mathematics that has concerned teachers and created anxiety in children. Problem solving has always been a favorite topic of persons doing research on elementary school mathematics instruction. In fact, there are probably more practical answers from research to help in the improvement of children's problem solving skills than for any other area of the elementary school mathematics curriculum.

How do pupils think in problem solving?

Studies by Stevenson (1925) and Corle (1958) reveal that pupils often give little attention to the actual problems; instead, they almost randomly manipulate numbers. The use of techniques such as "problems without number" can often prevent such random attempts.

What are the characteristics of good problem solvers? of poor problem solvers?

Researchers have identified a number of factors that are associated with high achievement in problem solving. Conversely, the lack of those factors is associated with poor problem solvers. Some of these traits are: intelligence, computational ability, ability to estimate answers, ability to analyze problems, arithmetic vocabulary, ability to use quantitative relationships that are social in nature, ability to note irrelevant detail, and knowledge of arithmetical concepts. (See: Engelhart, 1932; Stevens, 1932; Alexander, 1960; Hansen, 1944; Cruickshank, 1948; Chase, 1960; Beldon, 1960; Laughlin, 1960; Kliebhan, 1955; Butler, 1955; Klausmeier and Laughlin, 1961; Balow, 1964; Babcock, 1954.)

What is the importance of the problem setting?

Researchers such as Bowman (1929, 1932), Brownell (1931), Hensell (1956), Evans (1940), Sutherland (1941), Wheat (1929), and Lyda and Church (1964) have explored the problem setting. Findings are mixed, with some researchers suggesting true-to-life settings while others suggest more imaginative settings. While the evidence appears to be unclear, one thing does emerge: problems of interest to pupils promote greater achievement in problem solving. With today's rapidly changing world it seems unreasonable that verbal problems used in elementary school mathematics could sample all of the situations that will be important to pupils now and in adult life. Perhaps the best suggestion for developing problem settings is to take situations that are relevant for the child. Thus, a problem on space travel may be more "real" to a sixth grader than a problem based upon the school lunch program.

How does the order of the presentation of the process and numerical data affect the difficulty of multi-step problems?

Burns and Yonally (1964) found that pupils made significantly higher scores on the test portions in which the numerical data were in proper solution order. Berglund-Gray and Young (1932) found that when the direction operations (addition and multiplication) were used first in multi-step problems, the problems were easier than when inverse operations (subtraction and division) were used first. Thus, an "add-then-subtract" problem was easier than a "subtract-then-add" problem.

What is the effect of vocabulary and reading on problem solving?

Direct teaching of reading skills and vocabulary directly related to problem solving improves achievement (Robertson, 1931; Dresher, 1934; Johnson, 1944; Treacy, 1944; VanderLinde, 1964).

How does wording affect problem difficulty?

Williams and McCreight (1965) report that pupils achieve slightly better when the question is asked first in a problem. Thus, since the majority of textbook series place the question last, it is suggested that the teacher develop and use some word problems in which the question is presented first.

What is the readability of verbal problems in textbooks and in experimental materials?

Heddens and Smith (1964) and Smith and Heddens (1964) found that experimental materials were at a higher reading difficulty level than commercial textbook materials. However, they were both at a higher level of reading difficulty than that prescribed by reading formula analysis.

What is the place of understanding and problem solving?

Pace (1961) found that groups having systematic discussion concerning the meaning of problems made significant gains. Irish (1964) reports that children's problem solving ability can be improved by (1) developing ability to generalize the meanings of the number operations and the relationships among these operations, and (2) developing an ability to formulate original statements to express these generalizations as they are attained.

Should the answers to verbal problems be labeled?

While Ullrich (1955) found that teachers prefer labeling there are many cases in which labeling may be incorrect mathematically. For example:

Incorrect	Correct
10 apples	10
6 apples	+6
16 apples	16 apples

Does cooperative group problem solving produce better achievement than individual problem solving?

Klugman (1944) found that two children working together solved more problems correctly than pupils working individually. However, they took a greater deal of time to accomplish the problem solutions. Hudgins (1960) reported that group solutions to problems are no better than the independent solutions made by the most able member of the groups.

What is the role of formal analysis in improving problem solving?

The use of some step-by-step procedures for analyzing problems has had wide appeal in the teaching of elementary school mathematics. Evidence by Stevens (1932), Mitchell (1932), Hanna (1930), Bruch (1953), and Chase (1961) indicated that informal procedures are superior to following rigid steps such as the following: "Answer each of these questions: (1) What is given? (2) What is to be found? (3) What is to be done? (4) What is a close estimate of the answer? and (5) What is the answer to the problem?" If this analysis method is used, it is recommended that only one or two of the steps be tried with any one problem.

What techniques are helpful in improving pupils' problem solving ability?

Studies by Wilson (1922), Stevenson (1924), Washburne (1926), Thiele (1939), Luchins (1942), Bemis and Trow (1942), Hall (1942), Answers from Research: Problem solving (a-5b)

Klausmeier (1964), and Riedesel (1964) suggest that a number of specific techniques will aid in improving pupils' problem-solving ability. These techniques include: (1) using drawings and diagrams, (2) following and discussing a model problem, (3) having pupils write their own problems and solve each others' problems, (4) using problems without number, (5) using orally presented problems, (6) emphasizing vocabulary, (7) writing mathematical sentences, (8) using problems of proper difficulty level, (9) helping pupils to correct problems, (10) praising pupil progress, and (11) sequencing problem sets from easy to hard.

Note: There are many suggestions from research concerning verbal problem solving. It is suggested that the reader check the specific sources listed to further problem solving suggestions and for the representative material presented in the research articles.

Does teaching pupils to estimate improve achievement?

Dickey (1934) found that there was no difference in the achievement of groups who practiced estimation and those who didn't, while, in a better controlled study, Nelson (DA 1967) found that estimation was effective in increasing understanding of concepts. Faulk (1962) analyzed the estimates pupils made for a problem and found that only half gave acceptable responses. Corle (1963) found that fifth and sixth graders could estimate nearly as accurately as teachers and college students.

What are effective ways to teach estimation?

An analysis of the techniques for estimation presented in textbooks by Faulk (1962) revealed that finding sensible answers; estimating in computation to find sums, differences, products, quotients, measures, and averages; estimating answers to verbal problems; and rounding off numbers were all presented, but no one text treated all of these.

What effect upon pupil achievement does the teaching of mental computation have?

Improved ability to solve oral problems was reported by Flournoy (1954), while Petty (1965) found no significant difference between groups who did or did not use pencil and paper. Olander and Brown (1959) found that pupils had great difficulty when not allowed to use paper and pencil.

What techniques are best for improving mental computation?

A specified time allotment and step-by-step planned sequence of material seem necessary, according to Payne (DA 1966).

Wolf (1960) found that films and printed materials were equally successful as vehicles for presenting problems for mental computation.

Flournoy (1957) suggested the following experiences: (1) learning short-cuts for each operation, (2) practice in solving for both exact and estimated answers after listening to orally presented problems, (3) constructing problems, (4) learning to use rounded numbers, (5) realizing the importance of properly interpreting quantitative terms, and (6) learning to read and use graphs and tables. She noted (Flournoy, 1959) that about ten minutes per day should be spent on planned mental computation exercises.

Olander and Brown (1959) found that visually presented problems were easier than orally presented ones; this visual technique of presenting problems on flashcards was also used by Hall (1947) and Sister Josephina (1960). However, because pupils do not do well on something with which they have received little practice may indicate that they need more practice.

What is the role of mental computation in "new" mathematics?

This has not been answered by specific research; however, the general consensus seems to be that it should be included in any type of curriculum.

Does homework increase pupil achievement in elementary school mathematics?

Though assignment of homework is an accepted practice in many mathematics teaching situations, the value of homework is frequently questioned by teachers, parents, and pupils. Studies concerning the effect of homework on mathematic achievement are limited, and research on the effect of homework on achievement is confounded by a host of variables. Generally, the studies before 1960 do not show consistent results in terms of improved pupil achievement (Foran and Weber, 1939; Goldstein, 1960; Steiner, 1934; Teahan, 1935; Vincent, 1937). In a recent study, Koch (1965) found no difference in problem solving achievement, but significant improvement in concept achievement. Maertens (DA 1968) reported no significant differences between types of homework, as did Whelan (DA 1966).

What type of homework seems most effective?

Few studies investigate variables related to mathematics homework. Koch (1965) found that with sixth graders, both full or half-homework assignments resulted in significant achievement of arithmetic concepts. Steiner (1934) found arithmetic homework seemed to be more effective than English homework in terms of achievement. Slow sixth grade pupils showed greater gain than average pupils in a study by Vincent (1937). An individualized method was favored by Bradley (DA 1968).

Other questions: How much time should be spent on homework?
How should parents be involved in homework?

What type of review procedures should be used?

Intensive, specific review procedures were found to be effective by Burns (1960). He prepared lessons which included not only practice exercises, but also review study questions which directed pupils' attention to relevant things to consider.

Other questions: What effect on achievement does the use of review procedures have?

How much time should be spent on review?

When and how often should review occur?

How effective is checking as a procedure to reduce errors in mathematical problems?

Every mathematics teacher, at one time or another, has said, "Be sure to check your work when you have finished." Every student, at one time or another, has indeed checked his arithmetic problems for errors. And every student has been surprised to find that there were still errors in his work after checking. Grossnickle (1935, 1938) reported that checking is an ineffective procedure to reduce computational errors in division and subtraction. He found that if the student's check revealed some discrepancy, the student would force the check to that of the answer he produced for the problem. Karstens (1946) found that only in problems of estimation where a certain particular check was useful was any accuracy attained in checking procedures. It may very well be true that computational errors are mostly errors in understanding either the computational procedure or the underlying assumptions, or both. In that case, the check is another computational procedure to be misunderstood. Also, the lack of accuracy in checking may be related to the pupil's not sensing a reason to check.

Other questions: What per cent of the time do pupils check their work?
How can pupils be motivated to check?

How effective is the present teaching of numeral writing?

Little research has been done to answer this question in recent years. Hildreth (1932) found that the numerals 5, 8, and 2 were most difficult for kindergarten and first grade children to write, while 3, 9, and 7 were easiest. Newland (1930) reported that, for third through ninth graders, the order of illegibility was 5, 7, 2, 0, 4, 9, 8, 6, 3, 1. Two implications from most studies which are inherently sensible are that numeral writing must be taught or retaught at each grade level, and the need for legibility must be stressed throughout life. Buchanan (DA 1967) found that kindergarten pupils were able to learn to write numerals legibly, but this did not facilitate arithmetic conceptualization.

How can writing and reading numerals be effectively taught?

Most of the research which attempts to answer this question is found in the literature on the teaching of reading, since many of the same principles apply. Wheeler and Wheeler (1940) reported some success with the use of a game to teach children to read numerals, but this was under a rote teaching philosophy. The reading and writing of numerals is today connected more closely with the study for understanding of the decimal system.

How do children in the United States compare with children in foreign countries in elementary school mathematics?

There has always been a great deal of discussion regarding the performance of educational systems in foreign countries producing talented intellectuals, with the obvious implication that the educational systems throughout the United States do not. This was especially evident in the mid-1950's with the advent of Sputnik I. However, research studies directly comparing children from the United States and some foreign country are either rare or poor. Buswell (1958) reported that students in England in grades 5 and 6 were superior to students in the same grades in the United States. However, studies by Bogut (1959) and Pace (1966), using virtually the same data, concluded that this difference was attributable to the additional year of education that English children have up to that level. Johnson (1964) found that United States children were superior to English children as measured by an achievement test from the United States, while the opposite was true when the groups were tested by an English test. The attitude of American students was found to be more positive than that of London students by Johnson (DA 1966). Cramer (1936) revealed that United States children were superior to Australian children on an Australian test in grades 4 and 5, but just the opposite in grades 6 and 8. Wilson (1958) found no differences between United States and Canadian children in grades 2 and 3. Kramer (1959) found Dutch children superior to a group of children from Iowa. In general, children in foreign countries do as well as children in the United States as measured against their own standards, in their own culture, by their own measures.

Do foreign countries place a greater emphasis upon mathematics compared to the United States?

In order to evaluate emphasis, many researchers have examined and compared foreign textbooks, curriculum, and topic length. Sherman (1965) reported that the mathematics curriculum in Russia placed many mathematics topics at lower levels than the United States did and introduced more topics into the elementary school mathematics curriculum than the United States did. He concluded, however, that all of the topics were disconnected and discontinuous. Brownell (1960) reported on education in Scotland and concluded that children are able to handle mathematical topics earlier than now seems feasible, and that the attention span of children is longer than now thought. Miller (1960, 1962) reported for numerous European countries that elementary school mathematics topics, such as geometry, are introduced early in the curriculum, and that the textbooks contain more time for more rigorous practice. Dominy (1963), McKibben (1961), and Shutter (1960) all report that there seems to be little difference in achievement between countries. They also confirm the report that mathematical topics are introduced earlier in the curriculum than in the United States. Mehl (DA 1966) found that French textbooks placed more stress on problem solving.

What are the educational systems of foreign countries like?

There are many studies which discuss the elementary school mathematics programs of foreign countries without trying to compare them to that of the United States. They do not attempt to relate some of the advantages and disadvantages to the curriculum of the United States. They are merely reporting the ideas expressed or the materials used in the specific country. DeFrancis (1959) and Vogeli (1960) reported about Russia. Pella (1965) and El-Naggar (DA 1966) discussed the Middle East, while Zur (DA 1968) cited implications for Israeli. Buell (1963) examined Sweden. Sato (1968) studied Japan. Wirszup (1959) reported on Poland and other communist countries, as Fehr (1959) did with sixteen other European nations. Dutton (1968) studied the elementary school mathematics system of Ethiopia and recommended considerable changes be made in teacher education as well as in texts. Bruni (DA 1968) reported on recent Italian experimentation. There is a lot that can be learned in studying the educational systems of others in terms of ideas and materials use, but comparisons can be made only with extreme care.

Other questions: What materials and ideas used in foreign countries have proven to be very successful?

What concepts can be taught the pre-school child?

Few experimental studies have been conducted to answer this question. Some of the work of the Piaget-oriented researchers is correlated with this, and Sister Josephina (1964) reported success in teaching geometric shapes. Vast numbers of surveys have, however, been conducted to ascertain the mathematical concepts possessed by the child as he enters school. Considerable variability is found, both across and within the groups sampled, but it is generally concluded, as Woody stated in 1930, that young children do know a great deal of arithmetic. Rickard (DA 1967) and Heimgartner (DA 1968) recently reaffirmed this. Sister Josephina (1965) emphasizes the amount children learn incidentally, reporting that over 50% of her sample (aged 4 and 5) answered almost all items correctly. Williams (1965), however, reported success on his test for kindergarten children to be only 29%. The findings of the surveys are summarized by topic below; it should be noted, however, that sampling limitations restrict generalizability.

(1) Rote counting by ones: As MacLatchy (1931) noted, almost all kindergarten children can count. The mean was found to be 19 by Bjonerud (1960), while Brace and Nelson (1965) found most could count to 20 and Priore (1957) reported a mean of 29. MacDowell (1962) reported that three-year-olds could count to 5, four-year-olds to 30, and five-year-olds to 40. Earlier, Buckingham (1929), Brownell (1941), and Mott (1945) found that 90% could count to 10 and over half to 20, Stotlar (1946) found that 70% could count to 10 or higher, Wittich (1942) found all cound count to 10, and Woody (1931) stated that 25% could count to 100. MacLatchy (1930) reported the median for kindergarten children was 30 and for non-kindergarten children, 20.

(2) Rational counting by ones: The young child's ability to count by ones was found to be superior to that of counting by twos, fives, or tens (Brace and Nelson, 1965). The mean was found to be 19 for kindergarten children by Bjonerud (1960) and Buckingham (1929), while Brace

and Nelson (1965) found most could count to 20, though without knowledge of the structure of our numeration system. Holmes (1963b) reported that 84% could count to 10. MacLatchy (1930) and Woody (1931) reported that 70% of the kindergarten group could correctly enumerate 20 objects, while 92% could correctly select 5. Heimgartner (DA 1968) found that kindergarteners did better with number recognition in a series rather than in isolation.

(3) Counting by fives and tens: Bjonerud (1960), Brace and Nelson (1965), and Buckingham (1929) found that 25% of the kindergarten children tested could count by tens, while Priore (1957) reported that only 10% of her group could do so.

(4) Odd and even numbers: Less than 10% of the kindergarten children tested understood a sequence of odd numbers (Bjonerud, 1960).

(5) Ordinal numbers: Over 50% of the kindergarten children tested by Bjonerud (1960) had some understanding of ordinals, but Holmes (1963b) reported only 1% were successful with ordinal correspondence items. The relationship of counting to knowledge of ordinal number and place value was found to increase with age (Brace and Nelson, 1965).

(6) Grouping: All kindergarten children tested could recognize a group of three or fewer items immediately (Bjonerud, 1960). They used counting and grouping for larger numbers. MacLaughlin (1935) found that children recognized groups of up to four objects at age 5, counting on to find the number in larger groups. Recognition of groups of up to 5 in patterns and up to 4 in random arrangements differed from ability to recognize larger groups (Brace and Nelson, 1965; Douglass, 1925). Finding a subset with identical number properties was more difficult than matching sets with the same properties (Holmes, 1963a).

(7) Writing and reading numerals: Buckingham (1929) found that 85% of entering first graders could write to 5, 80% to 6, while Stotler (1946) said 26% could write numbers, Brownell (1941) reported 66% could write to 10, and Wittich (1942) reported 75% could. Priore (1957) found that 43% recognized all numerals through 10.

(8) Measurement: A large percentage of the kindergarten children tested were familiar with measurement terms and instruments (Bjonerud, 1960).

(9) Time: About 50% of the kindergarten children tested recognized time on the full hour (Bjonerud, 1960); Woody (1931) reported that about 25% could do this.

(10) Money: Mascho (1961) noted that entering grade 1 pupils seemed more familiar with money than with any other area of measurement. Bjonerud (1960) found that 80% of the kindergarteners recognized a penny, but only 38% recognized a nickel.

(11) Fractions: About half of the kindergarten children tested were able to recognize half of an item; 89%, thirds; and 66%, fourths (Bjonerud, 1960). In Priore's sample (1957), 78% recognized halves, 51% thirds; and 50%, fourths. Wittich (1942) reported comparable percentages for halves and fourths, but found few understood thirds. Woody (1931) indicated that about two-thirds of his sample had some knowledge of fractions.

(12) Geometry: Bjonerud (1960) found that the majority of the kindergarteners recognized circles and squares.

(13) Addition and subtraction: About 90% of the kindergarteners solved addition combinations in word problems; 75%, subtraction combinations (Bjonerud, 1960). Earlier, Buckingham (1929) found only 50% could correctly answer five of ten combinations, and MacLatchy (1930) reported that they knew a median of five combinations, with the use of objects increasing success. Priore (1957) found 34% could answer addition combinations and 50% were able to do most subtraction combinations.

(14) Vocabulary: Kolson (1963) reported that about 6% of the words kindergarten children use are arithmetic words.

(15) Other factors: Almost all studies noted a significant increase in achievement of concepts with age. No significant differences were found to be attributable to sex, nor to the effect of older children in

the family (Brace and Nelson, 1965). Socioeconomic status was found to be a factor by Brace and Nelson (1965) and Dunkley (1965). Grant (1938) analyzed the differences attributable to I.Q. levels of first graders.

Should topics for the pre-schooler be sequenced or incidental?

Roberts and Bloom (1967) reported no significant differences for type of program at the kindergarten level, as measured by tests of skills, concepts and general readiness. Dutton (1963) found that after a year in kindergarten without systematic instruction, 78% were above a norm he considered necessary for beginning systematic instruction.

What has been ascertained about readiness?

Brownell (1938, 1951) cited evidence of how children achieve to support his contention that children are ready to begin formal arithmetic instruction in grade 1. He recommends that abstract arithmetic should be translated into concrete experiences. In 1960, after comparing British and American schools, he added that children could learn more in the lower grades than we now ask. Dutton (1963) noted that 31% of the kindergarten children he tested were above the norm necessary for beginning systematic instruction in arithmetic. Koenker (1948) found that kindergarten pupils who had a readiness program achieved significantly higher gains on a readiness test than pupils who had a regular program.

Bruecker (1940, 1947), Hildreth (1935), Souder (1943), and Ferguson (DA 1967) are among those who reported on the development of tests specially designed to measure readiness. Kingston (1962) also cited correlations of readiness test scores with later achievement test scores, as did Olander, VanWagenen, and Bishop (1949). Binkley (DA 1967) also reported on correlations with personality adjustment.

Other questions: What is mathematical readiness?

Does there need to be a readiness stage for all topics?

What activities can be used to prepare the pupil?

How long is the readiness period?

The answers to these questions do not lie in the studies categorized for this topic; rather the answers are implied from other results. Many of the findings of the Piaget-oriented research in relation to developmental stages may have implications for ascertaining teaching stages.

What is the logical order of mathematical concepts?

Perreault (1957) identified stages in the process of identifying number groups: counting, partial counting, group counting, grouping, and multiplying.

The logical order of development of time concepts was noted by Ames (1946), while Ilg and Ames (1951) present the levels of attainment of the fundamental concepts and processes. The work of the Committee of Seven reported by Washburne (1928, 1931, 1936, 1939) is of course indicative of logical order.

Other questions:

- Is the logical order the same as the psychological order?
- Does teaching for meaning have an effect on order?
- Does teaching number properties have an effect on order?

What quantitative concepts do pupils have at each level of maturity and/or grade level?

Specific answers to this question are difficult to find, since it has rarely been asked as the focal question in a research study. However, it is implicit in most of the studies reported, and more specific topics should be searched for the answer.

In general, it has been found that qualitative understanding is closely related to the other achievement factors of computation, reasoning, and vocabulary; to reading ability and to intelligence (Muscio, 1962; Hall, DA 1967). Rappaport (1958) also found that computational skill did not indicate understanding of meanings. Significant relationships were found between a child's ability to conserve, seriate, and classify, and his level of achievement (Robinson, DA 1968). (Other Piaget-oriented research might also be applicable, as would the achievement evaluation studies.)

What content is appropriate for each grade level?

Such findings as these indicative ones are cited in studies in the following section:

(1) Study means, modes, and medians in grade 4 (Burns, 1963).

(2) Introduce geometric concepts and point set topology in grade 6 (D'Augustine, 1964) and geometric construction in grade 5 (Denmark and Kalin, 1964).

(3) Study other numeration systems in grade 1 (Scott, 1965) or grade 4 (Lerch, 1963).

(4) Study logic in grade 5 (Suppes and Binford, 1965).

The Committee of Seven studied placement of topics throughout the arithmetic curriculum, and made specific suggestions for the grade placement of topics which were accepted by many schools and textbook publishers (Gillet, 1931; Raths, 1932; Washburne, 1928, 1931, 1936, 1939).

When should formal instruction in arithmetic begin?

Related to the readiness question, this has been of much concern over the years, and has been explored in some of the more recent individualized instruction studies. Postponing formal instruction until grade 5 was concluded by Sax and Ottina (1958) or grade 6 by Benezet (1936). Brownell (1960) concluded that we should begin in first grade and teach more. Neureiter and Wozencraft (1962) are among those who explored the effect of removing grade level restrictions, reporting that this resulted in higher achievement and greater interest.

What is the most effective use of class time in elementary school mathematics?

Well-designed studies by Shipp and Deer (1960), Shuster and Pigge (1965), Pigge (1966), and Zahn (1966) reveal that maximum achievement in computation, problem solving, and mathematical concepts is obtained when over half of the time devoted to mathematical instruction is given to developmental teaching as opposed to practice. These studies reveal that pupils spending 75% of their time in developmental work were superior in all phases of elementary school mathematics compared with pupils spending 75% of their time in practice work. Hopkins (DA 1966) also supported this contention.

What is the difference in time spent on elementary school mathematics in other countries?

Miller (1958, 1960, 1962) has found that schools in foreign countries usually spend more time in the study of elementary school mathematics than do schools in the United States. Mathematical topics are introduced at an earlier level in most schools in Europe.

Is there an optimum amount of time that should be spent in elementary school mathematics? (Does the amount of time vary from grade to grade?)

Jarvis (1963) found that a period of 55-60 minutes produced substantially better performance than periods of 35-40 minutes. However, there is a lack of evidence in general regarding absolute amounts of time necessary to produce maximum achievement. Lawson (DA 1966) reported that fundamental skill scores were higher for a 60-minute regular group or a 40-minute concentrated group.

When should instruction in counting begin?

Woody (1931) noted that children had a considerable knowledge of counting before formal instruction began (at grade 2 for his groups). This has been supported by studies with pre-school, kindergarten, and first grade children (see the materials on "pre-first-grade concepts"). Woody also reported that only 2% of the parents indicated that they did not teach their children to count.

Most low-intelligence fourth graders could count by 2's; those in the average group could count by 3's to 16's; and in the high group, children could count by 3's to 23's (Feldhusen and Klausmeier, 1959).

Should pupils be taught rote or rational counting first?

Few studies have been directed at this point. Beilin and Gillman (1967) reported that number language knowledge is related to the cardinal-ordinal number task, but do not determine which comes first.

How much emphasis should be placed on sets before beginning to teach counting?

Studying kindergarten children Carper (1942) reported that the amount of grouping decreased and counting increased as pictorial context was increased. Dawson (1953) found that the greater the complexity and size of groups, the more counting occurred. Children aged 8 to 10 could only grasp a set of 4 objects, while those aged 10 to 12 could grasp 5 (Freeman, 1912). Various grouping patterns were studied by Brownell (1928), who reported that recognition of groups of dots was related to the size of the group but no numbers from 3 to 12 were more difficult. Children counted at first, then proceeded to more mature methods.

What techniques are most effective for teaching counting?

Dawson (1953) reported that geometric presentations might precede pictorial forms.

Should inequalities come before, after, or at the same time as equalities?

Holmes (1963) reported that finding a subset with identical number properties was more difficult than matching sets with the same properties.

When should formal number properties be taught? How should they be taught?

At grade 7 it was found that basic properties of addition were not clearly understood, with the distributive property apparently most difficult (Flournoy, 1964). Hinkelman (1956) found that only three of ten fraction principles were known at grade 5, while four were known at grade 6. Attainment of the concepts of commutativity, closure, and identity was found difficult for pupils in grades 2 and 4 by Bauman (DA 1966), while Schmidt (DA 1966) reported instruction on the commutative, associative and distributive properties was effective at fourth grade level. Gravel (DA 1968) showed that certain relations could be taught at grade 6.

What is the difficulty level of the various addition combinations?

MacLatchy (1933) found that (1) the easiest combinations were those in which 1 is added to a larger number, (2) a combination and its reverse form were not of equal difficulty, (3) adding a smaller number to a larger number was easier than the reverse form, and (4) combinations which contain a common addend were not of equal difficulty. Wheeler (1939) developed a rank order of difficulty of addition facts. It should be noted that these studies occurred before the extensive teaching of the commutative and associative properties. In programs where number properties are emphasized, the difficulty of combinations may be different than that reported above. At the present time studies using computer-assisted instruction are being conducted concerning the difficulty of basic addition and subtraction combinations. These findings may add to the pool of knowledge concerning this topic.

How can addition facts be effectively taught?

Researchers have found that (1) pupils with good counting facility learn addition facts effectively (MacLatchy, 1933); (2) teaching addition and subtraction facts together may result in higher achievement (Buckingham, 1927); (3) corrective work results in score-improvement on tests of basic facts (Wilson, 1954); (4) teaching addition combinations "indirectly" (practice within examples) rather than "directly" (in isolation) results in superior achievement (Breed and Ralston, 1936); (5) independent work improves mastery of the addition facts (Wilburn, 1945); and (6) use of simple manipulative materials increased understanding more than use of only pictures (Ekman, DA 1967).

What readiness should occur before formal addition is introduced?

MacLatchy (1932) found that pupils who were proficient in counting tended to have greater success in formal addition. There are other findings that point to the importance of developing counting and the ability to recognize the number of a set as good background experiences preceding addition. Also, Brownell (1928) found that thorough understanding of concrete numbers resulted in transition to abstract number with less difficulty and that difficulty with additive combinations were results of immature methods or lack of understanding of the relationships between experience with concrete and abstract.

Does checking answers result in improved achievement?

Clark and Vincent (1926) found that checking answers resulted in greater accuracy, especially when the number of problems attempted was considered. Thus, the technique of giving pupils fewer exercises, but having them check their answers, is suggested.

What procedures improve achievement in column addition?

Buckingham (1927) found that children taught to add columns downward achieved higher scores than those taught to add upwards. Ballenger (1926) found that dividing a column into two parts and adding each separately resulted in greater accuracy for pupils who could not achieve accuracy on longer columns.

How do pupils think when performing higher-decade addition?

Flournoy (1956, 1957) found that (1) the majority of children first noted the basic addition fact ending when performing higher-decade addition, recording first ones, then tens; (2) when bridging was involved, the carrying method was most frequently used; and (3) some children used different methods for horizontal and vertical forms.

How does socio-economic status effect pupil learning of addition facts?

MacLatchy (1930) found the rural children were less familiar with most of the addition combinations. Obviously further exploration needs to be done to obtain findings concerning the rural or urban child of today.

Other questions: When should the formal operation of addition be introduced?

Should addition be taught first in relation to union of sets or to counting?

How should renaming be taught?

When should the mathematical properties of addition be taught?

What type of subtraction situation should be used for introductory work? (how many more needed--additive, take-away, or comparison)

In an excellent study, Gibb (1956) found that the highest degree of pupil attainment was on take-away problems and the lowest level on comparative problems. She also found that additive problems took a longer time to complete. Schell and Burns (1962) found no differences in performance on the three types of subtraction problems. However, take-away problems were considered by pupils to be easiest.

Coxford (DA 1966) found that the procedure based on removal of a set from a set with no explicit use made of the relationship between subtraction and addition led to greater immediate proficiency than the more explicit procedure. Osburne (DA 1967) reported that a set-partioning-without-removal approach resulted in greater understanding than the take-away approach.

What are effective methods of teaching subtraction facts?

Gibb (1956) found that pupil performance was better on subtraction problems in semi-concrete context than in concrete context and lowest in abstract context. This suggests that pupils should be given wide semi-concrete and concrete experiences before proceeding to learn the subtraction facts. Buckingham (1927) found that pupils learned subtraction facts slightly more easily when they used a subtractive method rather than an additive method.

How should renaming in subtraction be taught?

Over the years researchers have explored procedures for teaching renaming (borrowing) in subtraction. Four different (or partially) different methods have often been explored. They are (1) take-away-renaming (decomposition), (2) take-away-equal additions, (3) additive-renaming (decomposition), and additive-equal additions. They are explained below.

Take-away-renaming

$$\begin{array}{r} 84 \\ \underline{-56} \end{array} \qquad \begin{array}{r} 80 + 4 \\ \underline{50 + 6} \end{array}$$

Six cannot be subtracted from 4.

$$\begin{array}{r} 70 + 14 \\ \underline{50 + 6} \end{array}$$

Rename.
Think 14 minus 6.
Think 70 minus 50

Take-away-equal additions

$$\begin{array}{r} 84 \\ \underline{-56} \end{array} \qquad \begin{array}{r} 80 + 4 \\ \underline{50 + 6} \end{array}$$

Six cannot be subtracted from 4.

$$\begin{array}{l} 80 + 14 \\ \underline{60 + 6} \\ 20 + 8 = 28 \end{array}$$

Add 10 ones to 4.
Add 1 ten to 50.
Subtract.

This procedure is based on the principle that if both terms are increased by the same amount, the difference (remainder) is unchanged. This property is referred to as compensation.

$$\begin{array}{r} 6 \\ \underline{-3} \\ 3 \end{array} \qquad \begin{array}{r} 6 + 2 \\ \underline{3 + 2} \end{array} \qquad \begin{array}{r} 8 \\ \underline{-5} \\ 3 \end{array}$$

Additive-renaming

$$\begin{array}{r} 84 \\ \underline{-56} \end{array} \qquad \begin{array}{r} 80 + 4 \\ \underline{50 + 6} \end{array}$$

$$\begin{array}{r} 70 + 14 \\ \underline{50 + 6} \end{array}$$

Rename.
Think 6 plus what number + 14?
Think 50 plus what number = 70?

Note that renaming is done in the same manner as in the classroom situation described above. The difference is in using "additive thinking" rather than "take-away" thinking.

Additive-equal additions

$$\begin{array}{r} 84 \\ \underline{-56} \end{array} \qquad \begin{array}{r} 80 + 4 \\ \underline{50 + 6} \end{array}$$

Six cannot be subtracted from 4.

$$\begin{array}{r} 80 + 14 \\ \underline{60 + 6} \end{array}$$

Add 10 ones to 4.
Add 1 ten to 50.
Think 6 plus what number = 14?
Think 60 plus what number = 80?

In a classic study, Brownell (1947) teaching of borrowing with meaning was more effective in both the equal additions and decomposition method. He also found that rational decomposition was superior to equal additions when the criteria was understanding and transfer, while mechanical teaching using equal additions produced smoother and faster performance.

Other findings comparing methods of teaching borrowing are: (1) equal-additions procedures produced fewer errors than decomposition (Osburn, 1927); (2) the additive method resulted in greater accuracy while decomposition was faster (Beatty, 1920); (3) few children taught the equal-addition method continued to use it (Taylor, 1919); (4) equal-additions was more accurate and faster than decomposition (Roantree, 1924; Johnson, 1931); (5) decomposition was more accurate than equal-additions, and there was no difference in speed (Rheins and Rheins, 1955); (6) decomposition was more popular with teachers (Wilson, 1934). Overall, it is reasonably safe to say that the decomposition method develops greater understanding while the equal-additions method is slightly faster.

How does the use of "crutches" affect teaching renaming in subtraction?

Brownell, Kuehner, and Rein (1939) and Brownell (1940) examined the method $\begin{array}{r} {}^{7}\cancel{8}{}^{1}6 \\ -39 \\ \hline 47 \end{array}$ as a "crutch" to borrowing in subtraction and found a significant decline in errors when the crutch was used. All but a small percentage of the children gave up the crutch readily.

Other questions: Should subtraction be introduced at the same time as addition?

Are separate subtraction facts needed?

What is the comparative difficulty of basic multiplication facts?

It is often suggested that basic multiplication facts should be presented in order of difficulty. There have been several attempts to order the difficulty of multiplication facts. Wheeler (1941) presented a table of rank order of difficulty of multiplication combinations and Ruch (1932) also presented such a table. However, this evidence must be regarded in light of the fact that a different theory of instruction was prevalent at the time of these studies than at the present time.

What procedures are effective in learning basic multiplication combinations?

Brownell and Carper (1943) found that: children taught mainly by drill did not have complete meaningful learning at the end of grade 5, but did have accuracy; habituation was used more frequently with easy combinations than with difficult ones; there were no high correlations between rate and C.A. or achievement; a moderate relationship between I.Q. and accuracy existed in grades 3 and 4; and there were higher median scores for girls than boys in lower grades. Wilson (1931) found that both bright and dull children learned equally well. Fowlkes (1927) found that a method using printed materials with a little teacher comment was efficient in teaching basic facts.

Clemmons (1928) found that specific drill reduced the error rate of pupils and zero facts proved to be difficult. Harvey and Kyte (1965) found that a program of diagnosis and remediation was effective.

At what grade level should multiplication be introduced?

Not many years ago multiplication was first introduced in grade three. The present practice is to introduce multiplication in grade two. Earlier studies by Brownell (1943, 1944) indicate that children were ready for multiplication combinations in third grade and were successful in learning them, that progress in accuracy on multiplication combinations was greatest in the fourth grade, and that pupil knowledge of multiplication facts increased in fifth grade.

Should the equal-additions or the Cartesian product approach be used for introductory work in multiplication?

In a good study Hervey (1966) found that: (1) Equal-additions multiplication problems were less difficult to solve and conceptualize, and less difficult to select a "way to think about" than Cartesian product problems; (2) Cartesian product problems were more readily solved by high achievers in arithmetic than by low achievers, by boys than by girls, and by those with above average intelligence though this was not substantiated with data.

Can pupils use the distributive property?

Gray (1965) found that:

(1) A program of arithmetic instruction which introduced multiplication by a method stressing understanding of the distributive property produced results superior to methods currently in use.

(2) Knowledge of the distributive property appeared to enable children to proceed independently in the solution of untaught multiplication combinations.

(3) Children appeared not to develop an understanding of the distributive principle unless it was specifically taught.

(4) Insofar as the distributive property is an element of the structure of mathematics, the findings tend to support the assumption that teaching for an understanding of structure can produce superior results in terms of pupil growth.

Schell (1968) found that when third grade pupils were taught basic facts of multiplication and the distributive property, they learned to use distributive property in two lessons plus a review lesson. Distributive property items were more difficult than non-distributive property items. Pupils scoring high on non-distributive items performed well on distributive items, but low scoring pupils had more difficulty with distributive than non-distributive property items.

Hall (DA 1967b) found that stress on the commutative property was effective with commuted combinations.

Is the subtractive or the distributive approach most effective?

During the 1940's and 1950's the distributive approach to division was typically taught in elementary school mathematics textbooks. With the beginnings of "modern mathematics" the subtractive approach became much more popular. The two approaches can be contrasted below.

Distributive

20 r8				
23 ⟌468	Think: "How	23 ⟌468		
46	many 23's in	238	10	Use any
8	400?" etc.	238		reasonable
		200	10	estimate
		8	20 r 8	

Dawson and Ruddell (1955) report that the use of the subtractive concept resulted in significantly higher achievement on immediate and delayed recall tests. They also found that a greater understanding of division and its interrelationships with other operations resulted from the study of division using the subtractive concepts and manipulative materials.

What is the role of "measurement" and "partition" division in the learning sequence?

Measurement division involves problems of the type: If each boy is to receive 3 apples, how many boys can share 12 apples?

Partition division involves problems of the type: If there are 4 boys to share 12 apples, how many will each receive?

Zweng (1964) studied measurement, partitive, and rate concepts of division, finding the partitive division problems were significantly more difficult than measurement problems. She also reported that rate problems may be easier than basic problems and partitive problems were more difficult than both basic measurement and rate measurement problems. Scott (1963) made use of two algorithms for division, using the subtractive algorithm for measurement situations and the distributive algorithm for partitive division situations. He suggested that: (1) the use of the two algorithms neither confused nor presented undue difficulty for young children; (2) teaching children to use two algorithms demanded no more teaching time than teaching only one algorithm; (3) children taught both algorithms had a greater understanding of the division operation than those taught by only one algorithm.

What is the most effective method of teaching pupils to estimate the quotient?

For early work in estimation of the quotient in division, two suggestions are usually made. There is the "apparent" method which suggests that the pupil look at the first digit of the divisor and the "increase-by-one" or "round-up" method in which the pupil is to increase the first digit of the divisor by one; thus 32 would become 40. Grossnickle (1937) found that:

(1) There were no significant differences between groups learning the apparent and the increase-by-one methods of quotient estimation, on either correct or estimation scores.

(2) There was no significant difference in the mean number of computational errors made when using the two methods.

While little research has been conducted to test the best method of estimating as far as pupil achievement is concerned, a number of studies have been conducted on the efficiency of various procedures. Morton (1947) analyzed 40, 014 examples and found that (1) the increase-by-one method is correct 79% of the time when the divisors end in 6, 7, 8, or 9; (2) the "apparent" method is correct 72% of the time when divisors end in 1, 2, 3, or 4; (3) for any divisor ending in 1 to 9, the apparent method is correct 53% of the time, and the increase-by-one method, 61%. Karstens recommends that the "second figure 5" divisors (25, 35, etc.) should be rounded upwards, since more correct trial divisors result.

Osburn (1950) analyzed division examples with divisors ending in 6, 7, 8, or 9, using a dichotomy, and revealed that the apparent method (Rule I) is successful in 4,800 cases where increase-by-one method (Rule II) is also successful; Rule I fails in 9,846 cases where Rule II is successful; Rule I is successful in 1,885 cases where Rule II fails; and Rule I fails in 2,099 cases where Rule II also fails. Osburn (1946) noted that the apparent method of estimating the quotient, with the instruction to try a quotient figure less by 1 when a subtrahend is too large, could enable the learner to handle all but 5% of any long division examples. Grossnickle (1931, 1932a, 1932b, 1939, 1945, 1946) also analyzed large numbers of division examples.

What are the difficulty level of division combinations?

Brueckner and Melbye surveyed to ascertain the difficulty levels of division. They reported that the sequence of difficulty from easy to hard is: (1) apparent quotient is true quotient (M.A. 10 to 11 years), (2) one-figure quotients, apparent quotient is not true quotient (M.A. 13 to 14 years); (3) two- and three-figure quotients with zeros (M.A. 13 to 14 years); and (4) two- and three-figure quotients, apparent quotient is not true quotient (M.A. 14 to 15 years).

Is it better to teach "long division" or "short division"?

"Long division" is the form

$$\begin{array}{r} 152 \\ 3\,\overline{)456} \\ \underline{3} \\ 15 \\ \underline{15} \\ 6 \end{array}$$

while "short division" is the form

$$\begin{array}{l} 3\,\underline{)\,4'56\,} \\ \quad 152 \end{array}$$

. Olander (1932) reports that most pupils chose to use long division. However, there was some preference for short division by good students. Grossnickle (1934) found that more errors were made by pupils using only short division. John (1930) also reports that the use of the long form was conducive to greater accuracy than was the use of the short division form.

Other questions: Should division facts be taught?

How can division be related to multiplication? to subtraction?

How can operations with fractions be taught effectively?

Fincher and Fillmer (1965), Traweek (1964), Greatsinger (DA 1968), Levin (DA 1968), and Wilson (DA 1968) found operations with fractions could be taught effectively by programmed instruction materials. Austin (DA 1966) reported both constructed and multiple choice formats were successful. Miller (1964) reported that written lesson plans plus automated practice machines were superior to use of the textbook plus concrete materials in teaching multiplication with fractions.

Krich (1964) reported that low I.Q. groups taught division with fractions meaningfully or mechanically did not differ in achievement, while the normal I.Q. group taught meaningfully scored higher on a retention test than a mechanically-taught group.

Gunderson and Gunderson (1957) found that second graders could understand fractions when they used manipulative materials. Audio-visual aids also helped fifth and sixth graders (Howard, 1950).

What is the best method for finding the common denominator for addition with fractions?

Anderson (DA 1966) found no differences for students using classes of equivalent fractions or factoring denominators when adding with unlike, unrelated fractions. Brownell (1933) evaluated the use of multiplication by the identity element to form a common denominator before adding with fractions. Labeling it a "crutch," he found children tended to drop it when simpler procedures were found.

What is the best method for teaching division with fractions?

Capps (1962, 1963) reported that the inversion method of teaching division with fractions was better for achievement on multiplication with fractions than the common denominator method, but on a retention test the inversion group remained at the same level while the common denominator group increased in achievement. Stephens and Dutton (1960) indicated that neither method was better on a retention test. Bergen (1966) cited evidence indicating the reciprocal and inversion methods were superior to the common denominator method. Bidwell (DA 1968) reported that the inverse operation method was superior to the complex fraction and common denominator methods in both structure and computational skills.

What is the best sequence for teaching division ideas?

Hirsch (1951) found that division with fractions was easiest when the division sign was used (2 3/4 ÷ 3 1/7). Next in order was "divide 3/4 by 5," followed by "divide 8 by 2 1/3."

What errors are commonly made when children compute with fractions?

Brueckner (1928) found that errors with fractions could be attributed to (1) computation, (2) lack of comprehension of the process involved, (3) inability to reduce fractions to lowest terms, and (4) difficulty in changing improper fractions to whole or mixed numbers. Shane (1938) found errors were caused by (1) difficulty in "reduction" in addition with fractions, (2) difficulty with "borrowing" in subtraction with fractions, (3) faulty computation in multiplication with fractions, and (4) use of the wrong process in division with fractions. Romberg (1968) reported that pupils using modern textbooks failed to cancel when multiplying with fractions more often than those using conventional texts.

Scott (1962) found that fifth graders made many more errors in subtracting with fractions involving regrouping than in whole number subtraction with regrouping, since pupils tended to relate the process to the decimal scale. Hinkelman (1956) found fifth graders knew an average of three of ten principles of fractions, while sixth graders knew four.

Diagnosis of errors in work with addition and subtraction with fractions did not seem to aid achievement, according to Aftreth (1957, 1958). Guiler (1936) used individualized group remedial work to improve scores on tests with fractions.

Other questions: What is the best physical world representation for fractions?

How should the various meanings that can be associated with the fraction (numeral) be taught?

What is the role of properties in teaching fractions?

Should addition or multiplication with fractions be taught first?

When, if ever, should the "cross-products" approach be taught?

What is the role of "mixed forms?"

Should addition and subtraction with fractions be taught together?

How can multiplication with fractions be given meaning, by arrays, grids, or addition?

Should all pupils be taught division with fractions?

How should decimals be related to place value?

In studying methods for placing the decimal point in the quotient, Flournoy (1959) found that multiplying by a power of ten was more successful than the subtraction method.

Other questions:

- When should decimals be introduced?
- Can decimals be developed with the metric system?
- What is the role, if any, of exponents in teaching decimals?
- What physical world or graphic devices are best for teaching decimals?
- How can division of decimals be "concretely" pictured?
- How should decimals and fractions be related?
- What concrete materials should be used in teaching decimals?

When should percentage be introduced?

Kenney and Stockton (1958) found that the three upper intelligence-level groups made significant progress in learning about percentage in grade 7; Kircher (1926) reported that only about one quarter of all pupils tested at grade 8 had acquired "an intelligent understanding." McCarty (DA 1966) reported success in teaching percentage at grades 4, 5, and 6.

How should "cases" of per cent be taught? (related or unrelated)

Guiler (1946) reported difficulty levels at ninth grade: finding a per cent of a number, 51.6%; finding what per cent one number is of another, 47.7%; finding a number when a per cent of it is known, 94.0%; finding the result of a per cent increase or decrease, 72.2%; finding a per cent of increase or decrease, 88.2%. Tredway and Hollister (1963) reported that teaching the three cases of percentage as related parts of a whole process provided for better retention.

What method should be used in teaching per cent? (ratio, unitary analysis, equations, formulas, decimals)

Wynn (DA 1966) found no significant differences in achievement or retention between unitary analysis, formula, or decimal methods.

Can per cent be effectively taught in the context of science and social studies?

Reavis (1957) found a project on stocks and bonds was effective; and Riedesel (1957) noted that most textbooks then currently in use had 1 to 4 pages on discounting of bank loans.

Answers from Research: Percentage (c-6)

Other questions: How can the "language of per cent" be effectively taught?

Answers from Research: Ratio and proportion (c-7)

How early in the grades can ratio and proportion be effectively introduced?

McCarty (DA 1966) reported success in teaching ratio at grades 4, 5, and 6.

Other questions:

- Should pure ratio and proportion (comparing like quantities) or rate pairs (comparing unlike quantities) be taught?
- What concrete materials are most effective for introducing ratio?
- How should ratios be related to other meanings for fractions?
- How should the "cross-products" method be taught?
- How can the identity element for multiplication be most effectively used in teaching ratio?

What should be the grade placement of concepts of measurement?

Many studies present the levels at which various time concepts are attained: Ames (1946), Friedman (1944), Harrison (1934), MacLatchy (1951), Spayde (1953), Springer (1951, 1952). Anselmo (DA 1967) reported positive relationships between time concept scores and I.Q., M.A., and C.A., but not S.E.S., while Tom (DA 1967) found I.Q. was not so important. Washburne (1939) reported data for the Committee of Seven on linear and square measures and time. Estimation of time was also of concern to Gilliland and Humphreys (1943) and Goldstone, Boardman and Lhamon (1958). Dutton (1967) was one of the few who experimented with teaching time concepts; he concluded that time concepts must be specifically taught to culturally disadvantaged children. In another experiment, Scott (1966) found that measurement terms in problems are not too difficult for intermediate graders. Eroh (DA 1967) found a structured program was better.

In other studies, size estimation was found to be affected by the value children gave objects (Blum, 1957), and by rewards (Lambert, Solomon and Watson, 1949). Very little change in size constancy occurred from ages 5 to 12 (Cohen, Hershkowitz and Chodack, 1958; Long, 1941). The greatest discrepancy between measurements and estimations was found to occur in weights and the smallest discrepancy in temperatures, with boys found to be more accurate than girls (Corle, 1960). Piaget-oriented research concerned with transposition and the size-weight illusion is reported elsewhere.

Paige and Jennings (1967) noted the inconsistencies of measurement content between first and second grade textbook series; greater agreement is found after grade three.

What materials are most effective for teaching measurement?

Programmed instruction was found to be effective in teaching area concepts (Keisler, 1959), but no different from traditional instruction for teaching latitude and longitude (Spagnoli, 1965). Students using S.M.S.G. materials achieved superior growth on measurement concepts, according to Friebel (1967).

Other questions: Should non-standard measurement precede the teaching of standard measurement?

Is measurement most effectively taught as a portion of mathematics or as a portion of the science-social studies curriculum?

Questions: When should integers be introduced in the grades?

What concrete materials are most effective in teaching integers?

Can operations with integers be effectively taught? At what grade level?

What are effective techniques of "rationalizing" integers?

Research on these questions is still minimal.

What is the effect of teaching algebra?

Braverman (1939) noted that algebra instruction resulted in increased arithmetic scores. Cassell (1963), reporting on the effect of S.M.S.G. instruction, noted increased scores in both arithmetic and algebra. No significant differences between programmed or traditional materials on equations and inequalities were found by Kalin (1962), and Messler (1961) found no significant differences after an algebra course. However, Banghart, McLaulin, Wesson and Pikaart (1963) found that, on a comparison of a traditional program and a modern mathematics program which included algebra, the modern program resulted in higher achievement scores.

Other questions:

- What aspects of algebra can be effectively taught in the grades?
- What aspects of algebra can be related to the problem solving program?
- What is the role of the axioms of equality in teaching in the grades?
- Can group and field properties be taught to pupils? If so, how can they be handled "concretely"?

What geometry can be effectively taught in the grades?

D'Augustine (1964) identified the following as highly teachable via programmed instruction: interior, exterior and boundary points; congruency; simple closed curves; triangle properties and definition; collinearity; finite and infinite points; and properties of lines and line segments. Weaver (1966) reported on an inventory for geometric understanding; he found no significant differences between conventional and modern classes. Instruction in coordinate geometry was reported effective by Herbst (DA 1968) at fifth grade level, and St. Clair (DA 1968) taught symmetry.

How can the vocabulary of geometry be most effectively developed?

Shepard and Schaeffer (1956) noted the knowledge of the name of an object helped pupils to achieve on a discrimination task.

What is the best sequencing of geometric topics?

Gagne and Bassler (1963), in connection with building a hierarchy, found that the group having the smallest variety of task examples in non-metric geometric materials retained less.

Other questions: Should two-dimensional or three-dimensional geometry be taught first?

Does the teaching of the notation of sets improve pupils' understanding and ability to deal with numbers?

Smith (DA 1968) found that students who received instruction in set theory showed significant superiority in logical reasoning.

What contribution do sets make to number operations? to geometric understanding?

The answers to these questions have only indirectly been attempted. Dawson (1953) found that the size and complexity of a group determined whether a child would count or would correctly identify the number of the set. Suppes and McKnight (1961) present suggestions for a first grade program, with written tests shown to result in higher achievement than teaching machine tests.

What materials are most effective in teaching ideas of logic?

The WFF'N Proof game aided logic scores (Allen, 1965), as did the S.M.S.G. program (Scott, 1965) and a program by Suppes (1964; Suppes and Binford, 1965).

Is there transfer from the teaching of logic to the teaching of problem solving?

The study of logic resulted in greater ability to verbalize mathematic generalizations, according to Retzer and Henderson (1967), but no research specifically related to problem solving was found.

Other questions: Can the notation and operations with logic be developed through sets?

How much formal logic can be effectively taught in the grades?

What materials are most effective in teaching place value?

Lyda and Taylor (1964) found that instruction on modular arithmetic did not result in greater understanding of our numeration system than the regular program did. Pupils who were taught place value concepts through the use of a ruler achieved a median retention score of 70% (Johnson, 1952).

What are the most common errors made by pupils?

In a survey, Flournoy, Brandt and McGregor (1963) found that errors related to (1) the additive principle; (2) relative interpretations; (3) the meaning of 1000 as 100 tens, 10 hundreds, etc.; (4) expressing powers of ten, as 10,000 = 10 x 10 x 10 x 10; and (5) the 10-to-1 relationship in place value.

Other questions: How much transfer to work with multi-digit number operations does the teaching of place value have?

What is the role of exponents, expanded notation and place value frames in teaching our notational system?

Is there transfer from historical systems to better understanding of our system?

Bradley and Earp (1966) found that few teachers stress underlying principles. Schlinsog (DA 1966) reported no significant effects of instruction on other number bases, while Scrivens (DA 1968) found that teaching about Egyptian numeration was more effective than teaching about base five numeration. Smith (DA 1968) reported that study of non-decimal systems produced effective achievement and retention, but little effect on decimal system understanding was found.

What are the most effective methods of teaching other bases?

Use of a variable base abacus was not found to result in greater achievement than use of the chalkboard (Jamison, 1964). A story about the use of a number base among a mythical group of people was effective according to Lerch (1963).

How much transfer to base ten does the teaching of other bases have?

Lerch (1963) reported that increased understanding of base ten resulted from teaching base five. However, in a very carefully conducted study, Schlinsog (1968) examined the effects of non-decimal instruction on basic understanding, computational ability, underachievement, and preference, and found no significant differences from regular decimal-base instruction.

Hebron (1962) did a factorial study of items and found that knowledge of one system is the most important single factor in learning a new one. Jackson (DA 1966) reported that pupils receiving instruction in non-decimal numeration systems did significantly better in tests measuring understanding and problem solving skills but not on computation than those studying the decimal system.

At what grade level can other bases be most effectively introduced?

Scott (1963) reported that first graders outperformed kindergarteners. Lerch (1963) and Hollis (1964) reported successful use of other bases in grade 4.

Other questions: What amount of time is efficient for teaching other systems of numeration?

What ideas concerning measures of central tendency can be developed?

Burns (1963) found that understanding of the mode and the mean could be taught in grade 4, while the median was a more difficult concept.

What concepts of probability can be effectively taught?

Probability learning was found to occur from the environment and was maximized by rewards (Messick and Solley, 1957). Smith (DA 1966) reported that topics in probability and statistics could be taught to most seventh graders. Ojemann, Maxey, and Snider (1965) found that third graders learned to make predictions when proportions were known, seeking more information before making predictions.

Other questions: What is the best grade placement of probability and statistical topics?

How can work with coordinates be most effectively taught?

What sampling ideas can be developed?

How are these concepts best developed?

How have textbooks changed over the years?

An extensive analysis of fifty-nine arithmetic textbooks for 150 years of publication (1790 to 1940) was done by Smith and others (1942, 1943, 1945). Basic changes that occurred in textbooks over the years were the inclusion of inductive method, increased "real life" emphasis, increased importance of learner interest, and change in content from emphasis on subject matter to meeting needs of user. In an analysis of teacher texts and student series, Hicks (1968) found a wide diversity of topics with less agreement on relevant topics for teacher texts than for pupil texts. Dooley (1960, 1961) studied the relationship of research to content on twelve topics, finding that clear, concise, exact recommendations were incorporated into textbooks within five years. Others used textbook analysis to ascertain the amount of content for specific topics.

How effective are modern mathematic textbooks?

The impact of S.M.S.G. materials on seventh, eighth, and ninth grade achievement has been investigated and reported in the literature. A very good study by Williams and Shuff (1963) compared programs using S.M.S.G. and traditional materials. They found that the seventh and ninth grade groups did not differ significantly in achievement gain. The only group that made any significant achievement gain was the eighth grade traditional group. Contradictory findings were reported in a study by Cassel and Jerman (1963) in that pupils of the same grade levels receiving S.M.S.G. instruction had statistically significant achievement when compared to students who had traditional instruction. Friebel (1967) found that the S.M.S.G. group achieved significantly more in arithmetic reasoning and on measurement concepts.

A related study by Nelson (1965) investigated the achievement of high ability pupils who used high or low level S.M.S.G. textbooks.

Generally, there were no significant differences in terms of the textbook used, but the high-ability, low-achieving students tended to perform better when using the lower level S.M.S.G. materials.

Hungerman (1967) found that groups taught with S.M.S.G. materials in grades 4, 5 and 6 scored better on contemporary tests, while traditional groups scored better on traditional tests.

Hughes (DA 1968) found that S.M.S.G. materials had had a greater impact on post-1960 commercially published textbooks than other materials had had.

How do teachers use textbooks and teacher's manuals?

Folsom (1960) found that about half of the teachers she studied did not use the manual, but had all pupils use the textbook page. Little use of the concrete and semi-concrete materials suggested by the manuals were made. Teachers particularly liked the combined textbook-manual. Butt (DA 1967) suggested a list of criteria for writing and producing textbooks.

Other questions: What proportion of mathematics time can most effectively be spent with the textbook?

What is the present grade placement of topics in textbooks?

Is a single-textbook or multiple-textbooks approach most effective?

Are workbooks effective in increasing mathematical achievement in elementary school?

Durr (1958), in an extensive study of workbooks in grades four to eight, found workbooks to be an effective aid in mathematical achievement in grades four and five. There were no significant differences attributable to workbooks found in grades six and above. Andreen (1938) found wide variations in achievement, depending on the use that teachers made of workbooks. When teachers relied on the workbooks to do their teaching for them, very little gain in achievement was noted. Stutler (1962) found that examining pupils' workbooks was a measure of mathematical achievement. In general, the research indicates that where proper use of workbooks is practiced, mathematical achievement can be increased.

Other questions: Are multi-level workbooks more effective than single-level?

How do workbooks compare in effectiveness with teacher-developed worksheets?

What proportion of teachers use workbooks?

Does the use of desk calculators, games, etc., improve learning?

Betts (1937); Fehr, McMeen and Sobel (1956); and Triggs (1966) reported that use of a calculator for work with fundamental operations resulted in increased achievement scores. An abacus helped to produce better computation scores more than workbooks did (Earhart, 1964), while Jamison (1964) found no differences resulting from use of a large abacus, individual abaci, or the chalkboard.

Dawson and Ruddell (1955) found that manipulative materials seemed to aid achievement in division. Plank (1950) noted that Montessori materials seemed helpful for remedial work. Training with Dienes' attribute blocks was compared with use of the Greater Cleveland program by Lucas (DA 1967). He found that the attribute block group were (1) better conservers, (2) better at conceptualization of addition and subtraction, (3) not as good in computation, (4) no better on problems, and (5) slightly better at multiplication.

In general, such materials seem to be more effective for slow and average learners than for those achieving above average.

Does the use of Cuisenaire materials improve mathematical achievement and understanding?

Brownell (1963), after interviewing English children who had used the Cuisenaire program, reported that they responded more quickly to simple combinations than did traditionally-taught students, and used more sophisticated solutions for unknown combinations. However, the traditional group was more accurate. No differences were noted in understanding or problem-solving. In another study (Brownell, 1968), he found that Scottish students using the Cuisenaire program had less instruction time and demonstrated greater maturity of thought processes than conventional groups. The Cuisenaire group did not, however, perform better in verbalizing answers. For English students, conventional group ranked higher, with Cuisenaire and Dienes programs about equal on conceptual maturity. All three programs were similar for problem solving.

The Cuisenaire program taught traditional subject matter as well as the traditional method when measured by an achievement and a traditional test, according to Hollis (1965). Additional concepts and skills were acquired by the Cuisenaire pupils. Nasca (1966) added evidence to support this. Lucow (1963, 1964) reported that the Cuisenaire program was as effective for third graders as the traditional program in teaching multiplication and division. On the other hand, Passy (1963, 1964) found that third grade children using Cuisenaire materials achieved significantly less than other groups.

Fedon (DA 1967) noted that maximum manipulation was the essential factor, and first graders using Cuisenaire materials achieved slightly less well than those using an eclectic approach. Callahan and Jacobson (1967) found that the rods could be used effectively with retarded children.

Are teacher-made materials effective?

Harshman, Wells, and Payne (1962) found that teacher-made materials were as effective as either expensive or inexpensive purchased materials. In contrast, Reddell and DeVault (1960) reported that two commercial aids increased some aspects of achievement more than teacher-made aids did.

Who should manipulate materials, the teacher or the pupil?

The group using individually manipulated materials made greater gains than the group seeing only a teacher demonstration, according to Toney (DA 1968). Trueblood (DA 1968), on the other hand, reported that pupils who saw only the teacher manipulative materials scored higher than pupils who manipulated materials themselves.

Other questions: What is the optimum amount of time that should be spent in the use of concrete materials before the use of abstract symbolism is profitable?

How effective are mathematics laboratories?

What is the best "mix" of multi-media?

What multi-sensory aids are currently being used in school systems?

With what topics do audio-visual devices aid in teaching mathematics?

Suppes, Jerman, and Groen (1966) reported that practice on arithmetic facts can be presented via a computer-connected teletype. Anderson (1957) reported that use of a kit of visual-tactual devices was helpful in a unit on area and volume. Howard (1950) noted that retention for a group using audio-visual aids for fractions was better.

Many other studies used audio-visual devices, but did not explicitly test their effect.

How effective is television in teaching mathematics?

Television instruction did not seem better than conventional instruction, reported Jacobs and Bollenbacher (1960). It seemed more effective, however, when seventh graders were grouped homogeneously (Jacobs, Bollenbacher, and Keiffer, 1961). Kaprelian (1961) reported a more favorable attitude toward arithmetic by fourth graders as a result of a televised course. The "Patterns in Arithmetic" television course was noted by Weaver (1965) to be as effective as traditional course.

Other questions: How effective are films and filmstrips in teaching mathematics?

How can audio-visual devices be used most effectively?

How effective is programmed instruction in teaching mathematics?

Results from studies using various ways of presenting programmed material show differing results. Banghart, McLaulin, Wesson and Pikaart (1963), Brinkmann (1966), and Fincher and Fillmer (1965) found pupils having instruction via various methods of programmed instruction as compared to conventional instruction, made significant achievement gains, but Arvin (DA 1966), Donaldson (1968), Feldhusen, Ramharter, and Birt (1962), Meadowcroft (1965), and Spagnoli (1965) found no significant differences. Pupil attitudes were found to be more favorable toward programmed instruction by Feldhusen and others (1962), but Meadowcroft (1965) found accelerated pupils having more favorable attitudes toward a method using the least amount of programmed instruction, and Brinkmann (1966) found pupils who were below the median in achievement favored teacher instruction.

How effective are various methods of presenting programmed instruction?

Programmed instruction can be presented in a variety of ways. Eigen (1962) found no significant difference when materials were presented by teaching machines, vertical text, or horizontal text, and Higgens and Rusch (1965) found no differences for programmed textbooks versus a workbook for remedial teaching. Miller (1964) found written plans plus automated practice machines superior to textbooks with concrete materials in achievement gains. A study by Crist (1966) found no difference in individual or group-paced use of programmed texts.

Austin (DA 1966) found that both constructed responses and multiple choice responses were effective.

How can programmed instruction be most effectively used as part of the teaching process?

Of the infinite number of ways that programmed instruction could be used with teacher instruction, few combinations have been investigated and reported in research literature. Programmed instruction during teacher instruction as contrasted with preceding and following teacher instruction, was investigated by Meadowcroft (1965). This study found more positive attitudes for all groups, but higher achievement for the average group who had programmed material during teacher instruction.

What types of pupils seem to benefit the most from programmed instruction?

The use of programmed instruction with mentally retarded children (Blackman and Capobianco, 1965), resulted in significant behavior change but not significant achievement when compared to conventional methods. Kalin (1962) found that programmed materials did not produce superior achievement with high I.Q. pupils. However, less time was needed to finish materials. In contrast, Fincher and Fillmer (1965) found high I.Q. pupils performed better with programmed instruction. Traweek (1964) found no significant difference for I.Q., but concluded that programmed instruction may be a promising method of teaching poorly adjusted students.

What mathematical content has been taught with programmed materials in research situations?

Frequently an experimenter will select a topic that pupils would normally have little knowledge of, thus adding control in terms of the limited scope of initial knowledge. Geometry topics, including topology, sets, relations and functions, have been used by Brinkmann (1966), D'Augustine (1966), Denmark and Kalin (1964), Gagne and Bassler (1963), and Randolph (1964). Advanced topics were used by Kalin (1962) and latitude and longitude by Spagnoli (1965). Various operations with fractions were used by Greatsinger (DA 1967), Krich (1964), Levin (DA 1968), Miller (1964), and Traweek (1964), and Eigen (1962) used numbers and numerals. General lower grade arithmetic was used by Banghart and others (1963) and by Fincher and Fillmer (1965). Remedial multiplication and division was studied by Higgins and Rusch (1965). Riggs (DA 1967) developed a text to interpret graphs.

How effective is CAI in teaching mathematics? (How can CAI be effectively used?)

Suppes has reported (in various progress reports for the Stanford Project) success in using both drill and practice and tutorial computer-assisted instruction programs at the primary grade level.

What is the reading level of current mathematic textbooks?

Research indicates that many problem solving difficulties are actually reading difficulties. The assumption that a text for a certain grade is based on the reading level of that grade may be a false assumption. Buswell (1931) indicated this was a problem of concern many years ago, and recent research indicates the problem is still with us. Smith and Heddens (1964) found the reading level of experimental mathematic materials was usually above the grade level of use. They also found the same true of five commercial textbooks, with great variation between and within the textbooks. A study by Repp (1960) which may be relevant found 1379 or more new words introduced in third grade textbooks. It seems realistic to investigate the reading level and increase in new vocabulary when selecting textbooks, and not to make the assumption the text will be appropriate for the grade level. Covington (DA 1967) reported that the reading level of a series of modern texts was too difficult for third and fourth graders. Reed (DA 1966) found little agreement between vocabularies in reading and arithmetic texts.

What can be said about the specific vocabulary (technical language) used in textbooks?

The frequency of specific vocabulary in textbooks has been investigated by many researchers in the past. Of words occurring five or more times, Brooks (1926) found 237 and Gunderson (1936) found 252. A wide variation in the actual vocabulary or technical terms is found between textbooks (Pressey and Elean, 1932; Repp, 1960; Willey, 1942). Currey (DA 1966) reported that new terminology is confusing to low-socio-economic-level first graders. Stevens (DA 1966) found that between 1956 and 1964, the vocabulary load increased more than forty per cent.

What is the relationship between vocabulary and learning mathematics?

The ability of children to understand vocabulary or technical concepts varies greatly for individuals and generally increases with intelligence, achievement, age and grade (Brotherton, 1948; Chase, 1961; Cruickshank, 1946). When specific training in mathematics vocabulary is carried on, Dresher (1934) and Johnson (1944) found definite gain in vocabulary and ability to solve problems. Lessenger (1925) found general reading instruction improved problem solving. Both Hanson (1944) and Treacy (1944) found a close relationship between composite reading skills and problem solving ability. It would appear that reading ability of students, reading level of materials, and vocabulary of both must be considered as being closely interrelated with learning to solve verbal problems.

Other questions: How verbal should mathematics books be for effective teaching?

How can problem readers who have mathematical ability be most effectively taught?

What effects do quantitative concepts have upon other subject areas?

The most frequently used concepts of mathematics used in other subject areas are time, measurement, money, and distance. These concepts not only permeate the curriculum of other subject areas but also the environment of every pupil. It would not be desirable or even possible to confine such topics to a mathematics text or class. However, many pupils are penalized in English or social studies for not understanding the quantitative concepts that are included in those subject areas. Jarolimek and Foster (1959) found as many as four hundred quantitative concepts on a ten-page sample of one social studies text. Lyda and Robinson (1964) classified nine hundred concepts that were found in three social studies texts. Older studies by Partridge (1926) and Woody (1932) found similar concepts in English texts. After discovering the extent of the material contained in these sources the researchers attempted to measure the pupils' understanding of those concepts that were found. They found that only fifty per cent of the mathematical concepts found in English and social studies texts were understood by pupils using those texts. All of the researchers agreed that greater emphasis should be placed upon understanding of basic quantitative concepts taught in elementary school mathematics.

Other questions: Is the same vocabulary for quantitative terms used in other subjects and in mathematics?

What are the most common errors made by pupils? (What are the most common misconceptions that pupils have concerning mathematical understanding?)

It was generally agreed that errors with combinations were the most frequent source of error. In an extensive diagnostic study (Buswell, 1926), various poor work habits were cited for each operation; many of these, however, are related to the teaching procedure and are no longer completely appropriate. Nevertheless, errors with combinations were most frequently cited. Specific remediation based on diagnosis of the errors was found to be fairly successful.

Smith and Eaton (1939) found addition facts were most thoroughly mastered at the fourth grade level, with zero combinations most frequently missed.

In analyzing errors with fractions, Brueckner (1928a) found 21,065 errors, of which the major ones were computational. Lack of comprehension of which process was involved, inability to express fractions in simplest form, and difficulty in renaming improper fractions were also causes of error. Morton (1924) and Shane (1938) substantiated these results. Scott (1962) found regrouping errors with subtracting fractions were more frequent than in whole number work. More errors of this type were found with children using a contemporary program than Brueckner noted in 1928.

Brueckner (1928b) found 114 different kinds of errors with decimals; most common was misplacement of the decimal point. Guiler (1946a) reported that changing fractions to decimals, renaming mixed numbers, and division with decimals were the greatest sources of difficulty.

For addition and multiplication, Burge (1932, 1934) reported that errors with combinations and carrying were most frequent. Knight and Ford (1931) noted that the later a multiplication fact appeared in an example, the more frequent were errors with it, but Wilson (1936) disputed this.

Grossnickle (1934, 1935, 1936a, 1936b, 1939, 1941, 1943) analyzed division errors, reporting that combination errors were most frequent (38.8%), while difficulties with remainders accounted for almost one-fourth of all errors. Errors with zero facts were constant across all operations (Grossnickle and Snyder, 1939).

In work with percentage at grade 9, Guiler (1946b) reported that at least half of the pupils had difficulty, with almost everyone unsuccessful at finding a number when a per cent of it is known.

Lutes (1926) found that errors on verbal problems resulted primarily from computation, than from ignorance of a principle or rule, and finally from lack of comprehension. Morton (1925) reported that use of incorrect procedures accounted for over half of the errors. Errors with addition and subtraction in word problems were less frequent than those with other operations (Ross, 1964). Roberts (1968) analyzed third grade test papers, and categorized four types of errors: wrong operation, computational, defective algorithm, and undiscernable, with defective algorithms accounting for the largest number of errors.

How can errors be most effectively diagnosed?

Brownell and Watson (1936) and Burge (1934) reported that use of an interview technique was more reliable in ascertaining errors than a test was. Brueckner (1928a, 1928b) and Brueckner and Elwell (1932) counted errors with fractions and decimals made in written work. Grossnickle (1935) reported that he found that at least three responses to each fact must be made by pupils for diagnosis to be reliable. It was suggested by Olander (1933) that teachers diagnosed more accurately in division than in the other three processes. Aftreth (1957, 1958) reported that systematic analysis of errors in the study of fractions was not particularly helpful, while Dougherty (1962) presented a more successful program in which pupils diagnosed their own errors. Guiler

(1936) used individualized group remedial work; Harvey (1953) suggested specific provisions for reteaching. Eaton (1938) used a dictaphone to record verbal responses successfully.

What are the causes of low achievement in mathematics?

Bernstein (1956) indicated that both intellectual and emotional factors are relevant. Easterday (1964) identified (1) low ability, (2) psychological problems which prohibit a child from functioning at his level of ability, (3) insufficient motivation, (4) inability to read and comprehend written material, and (5) general discipline problems.

What procedures are effective with the pupil with problems in mathematics?

That planned remedial instruction improves achievement has been shown by many studies: Bemis and Trow (1942), Bernstein (1956b), Callahan (1962), Cooke (1931, 1932), Fogler (1953), Guiler (1929, 1936), Guiler and Edwards (1943), Tilton (1947). Such programs appeared to be especially effective when instruction was individualized, to meet specific, diagnosed needs. Lerch and Kelly (1966) reported that a seventh grade program planned for slow learners, with intensive teacher-pupil interaction, was successful.

Higgins and Rusch (1965) found that a programmed text and a workbook were equally useful for remedial teaching. S.M.S.G. materials were successfully used with slow learners, according to Easterday (1964).

What has been found about teaching mathematics to mentally retarded pupils?

Programmed instruction was successful when used with mentally retarded pupils (Blackman and Capobianco, 1965; Rainey and Kelley, 1967; Jenkins, DA 1968; Johnson, DA 1967; Pinegar, DA 1968). Callahan and Jacobson (1967) reported that use of Cuisenaire rods increased the understanding of retarded children in one class.

The context of a problem did not appear to affect the achievement of retarded children (Finley, 1962), but superfluous material in a problem caused difficulty (Cruickshank, 1948a). Naming one process and solving by another was more typical of retarded children than of normal children (Cruickshank, 1948b).

Gothberg (1949) found that not until the mental age of 5 could at least 50% of the mentally retarded children she studied respond to time percepts, with abstract concepts such as historical time not understood until at least a mental age of 10. Similar lags are reported across other topics. For instance, Quick (DA 1967) reported that the three stages of development described by Piaget do occur in order for the mentally retarded, but there is a lag.

Hoelte (DA 1967) reported that retarded children in a special class did not achieve more than retarded children in a regular class. The use of appropriate measuring instruments for such pupils has been a matter of concern: two tests have recently been developed for use with mentally retarded children (Connolly, DA 1968; Pritchett, DA 1966).

Other questions: What is the most important variable in handling individual differences--materials, sequencing, time, pacing, scope?

What procedures motivate the slow learner?

How can the curriculum be effectively varied for the slow learner?

What materials should be used in "center city?"

Has acceleration proven to be effective for the superior pupil?

Aftreth and MacEachern (1964) found that both an acceleration and an enrichment program were effective. Townsend (1960) and Ivey (1965) offered further evidence to show that acceleration is possible, even when not limited to those with high I.Q. scores. Jacobs, Berry and Leinwohl (1965) indicated that the effect of acceleration was observable only over a period of time.

Klausmeier and Ripple (1962) found no unfavorable academic, social, emotional, or physical correlates of acceleration from second to fourth grade. Matched control pupils who had been randomly assigned to non-acceleration achieved significantly less than those who were accelerated. In a follow-up study, Klausmeier (1963, 1964) found that the accelerated pupils were continuing to show no harmful effects and were achieving as well as bright children at the advanced level. Data from Rusch and Clark (1963) completely support the Klausmeier and Ripple findings at intermediate grade levels.

There is also some evidence to show that homogeneous grouping is especially effective for the upper ability group (Balow and Ruddell, 1963; Provus, 1960).

What strategies do gifted pupils use?

Namy (1967) found that gifted pupils and those misdiagnosed as gifted had similar achievement scores. The latter group apparently relied highly on memory in attaining knowledge.

What topics have proven effective with the superior pupil?

Kalin (1962) taught intellectually superior pupils a unit on equations and inequalities using both programmed instruction and conventional techniques, which were equally effective.

Lewis and Plath (1959) found that high ability children could develop generalizations about numerical relationships at a more advanced level than those normally presented to them.

As part of a long term project, Suppes (1966) and Suppes and Ihrke (1967) reported on the use of materials on sets, coordinate systems, geometry, signed integers, logic, and symmetry.

In general, few topics have not been found to be effective with bright students.

Other questions: What is the most effective "mix" of vertical and horizontal enrichment?

What grouping procedures have proven most effective in teaching mathematics? (How effective is homogeneous grouping?)

Homogeneous (ability) grouping was reported to result in favorable arithmetic achievement by Balow and Ruddell (1963), DeWar (1963), Echternacht and Gordon (1962), McLaughlin (1961), Pinney (1961), Provus (1960), Savard (1960), and West and Sievers (1960.

Difficulty in forming homogeneous groups was noted by Balow (1964).

Heterogeneous grouping was found to be more favorable for arithmetic achievement by Barthelmess and Boyer (1932) and Koontz (1961).

No differences between the two plans were reported by Davis and Tracy (1963), Holmes and Harvey (1956), Wallen and Vowles (1960), or Willcutt (DA 1967).

Individualized programs were suggested by Fawcett and others (1952), Graham (1964), Hamilton (1928), Jones (1948), Klausmeier (1964), Nabors (DA 1968), Nee (1939), Potamkin (1963), Redbird (1964), Sganga (1960), and Thompson (1941). Brewer (1966) found that teachers with "high" academic qualifications were more likely to realize the need to individualize. Availability of materials, awareness of the pupil ability range, interest, and time to plan were important factors for grouping.

How effective is Individually Prescribed Instruction (IPI)?

Generally studies show that achievement on standardized tests is about equal to that of conventionally grouped students, while progress on IPI tests and standards is satisfactory (Bartel, DA 1966; Deep, DA 1967; Fisher, DA 1968; Scanlon, DA 1967; Yeager, 1967).

Other questions: How can materials be most effectively used with varying patterns of grouping?

What are pupil attitudes toward grouping?

How do personality factors affect achievement?

Under-achievement has been related to personal adjustment and is often considered as influential in relation to achievement in arithmetic as intelligence is. Various aspects and degrees of adjustment have been investigated in relation to arithmetic achievement with some interesting results. A study by Capps (1962) found retardation in arithmetic tended to be related to personal adjustment, and positive correlations between arithmetic achievement and a health personality were found by Cleveland and Bosworth (1967). Wilson (1959) had contradictory results in that no certain differences in arithmetic achievement were found for pupils who scored at or below the tenth percentile on a personality test, when compared to pupils who scored at the 50th percentile. Ridding (1967) found extraversion correlated with over-achievement and introversion correlated with under-achievement. A related study by Buswell (1953) found, when intelligence was controlled, status of social acceptability was not related to achievement.

Children classified as emotionally disturbed were found to have lower arithmetic scores than reading scores in two studies, one by Stone and Rawley (1964) and one by Tamkin (1960).

The relationship of delinquency or social maladjustment to arithmetic achievement has been investigated in several studies. Socially maladjusted boys showed poorest achievement in the area of arithmetic (Feinberg, 1947) and delinquent below-average I.Q. children performed better on non-verbal intelligence tests than verbal intelligence tests (Richardson and Saerko, 1956). Dinitz and others (1957) found delinquent-prone boys had significantly less arithmetic competence than non-delinquent-prone boys, and an older study by Lane (1934) found delinquent boys poorest achievement was in subject areas which required drill, as in arithmetic computation.

Are there any sociological characteristics that distinguish pupils of varying mathematical ability?

In the past, the one-room schoolhouse was a common educational situation. In the 1930's, the relationship of achievement to the sociological characteristics of rural or community schools was investigated by McIntosh and Schrammel (1930) and Clem and Chester (1933). Both studies investigated achievement of rural schools as compared to village or graded schools, and both found the village or graded schools had higher arithmetic achievement.

Some older studies, concerned with cultural characteristics which are still prevalent in today's society, compared achievement of white children to achievement of Mexican children (Coers, 1935) and American Indian children (Hansen, 1937). The white children had higher arithmetic achievement scores in both studies, but when Coers considered mental ability, Mexican children were found to be achieving more for their measured level. A study by Manuel (1935) found Spanish-speaking children had higher arithmetic achievement scores than English-speaking children, but the reverse was true of reading achievement scores. Harris (DA 1968) reported that Negro children achieved less well than white children, but did better in arithmetic than in most other areas.

Does handedness have an affect on arithmetic achievement?

Various physical characteristics of pupils have been investigated to see if they differ with different levels of mathematics achievement. One physical characteristic that has been investigated, mainly in relation to reading, is handedness. Groff (1962) carried on an investigation of the relationship of hand preference to arithmetic achievement, and found some differences that indicated the left-handed pupils had lower reasoning scores. He points out other factors may have accounted for the differences found in his study.

Early elementary teachers are constantly aware of pupils reversing letters and numbers when writing. A study that deals with confusion or nondominant handedness as a possible explanation of reversals was done by Zaslow (1966). He found that having children move the hand and arm so it crossed the body in mid-line resulted in significant corrections of reversed numbers and letters.

To what extent do siblings resemble each other in intelligence and mathematical achievement?

A study by Schoonover (1956) found a substantial relationship between the intelligence and achievement of siblings, with intelligence having a higher correlation than achievement for siblings. He found that sisters were more similar to each other in arithmetic achievement than in other achievement areas.

What are the achievement characteristics of children from orphan or foster homes?

Two studies by Feinberg (1949, 1954) were concerned with achievement of children from foster and orphan homes as compared to achievement of maladjusted children. Children from foster homes achieved more than children from orphanages. Both groups achieved more than maladjusted children, but arithmetic was found to be a difficult subject for them as a group.

What differences in mathematical achievement can be attributed to sex?

It should be noted that differences related to sex are not limited to mathematical achievement in elementary school. There is also a distinct difference between pre-junior high achievement and achievement of those of junior high school and beyond. Almost all of the related research indicates that pre-junior high school girls achieve more than pre-junior high school boys except in arithmetic. Studies by Heilman (1933), Stroud and Lindquist (1942), Powell (1963), and Jarvis (1964) all support this indication and show no significant differences between the sexes in arithmetic achievement. From junior-high school and beyond the research indicates the same superiority of girls in general, but boys now surpass girls in studies involving science and mathematics. Studies by Blackwell (1940), Alexander (1962), Wozencraft (1963) and Powell (1964) support this view.

What differences in mathematical achievement are related to self-concept?

In many cases it is not as much ability that determines achievement as the student's concept of his ability: "How well should I be doing in relation to the other students?" There seems to be some indication in the studies by Unkel (1966) and more especially Clark (1967) that girls do not show superior achievement in arithmetic, science, and mathematics simply because they feel that girls should not show superior achievement in those fields. Boys, on the other hand, may feel that subjects other than science and mathematics are not masculine enough, not rough and ready enough, for them to show superiority. Certainly adjustments need to be made in the mathematics curriculum to accommodate girls as well as boys.

Can differences in mathematical achievement be attributed to differences in socioeconomic environment?

One of the most important topics for discussion in education today is the topic regarding socioeconomic environment and achievement. Studies by Montague (1964), Dunkley (1965), Dutton (1967), Binkley (DA 1967), Searle (DA 1968), Skypek (DA 1967), and Unkel (1966) all reveal that there is a high correlation between socioeconomic environment and achievement, and that the lower the level of socioeconomic environment the lower the elementary school mathematics achievement. This relationship seems to indicate that children from low socioeconomic backgrounds have a scholastic handicap in direct proportion.

Can differences in mathematical achievement due to socioeconomic differences be reduced?

It seems logical that to reduce the differences in mathematical achievement due to differences in socioeconomic environment would be to reduce the differences in the socioeconomic environment. Since this seems to be impossible, attempts have been made to reduce the effects of the environment. Paschal (1966) and Newman (1967) found that by recognizing the handicap that a low socioeconomic environment places on a pupil they could, by paying special attention and giving great amounts of individual assistance, reduce the differences in achievement by increasing the achievement of these individual pupils. Pitts (1968) found more success in reducing the environmental handicap by providing preschool experience to as many of the children from low socioeconomic background as possible. This program was similar to the Project Head Start. Hollander (DA 1968) reported gains in speed and accuracy when verbal praise and candy rewards were given to sixth grade inner city children.

What procedures are most effective in testing computational skills? understanding?

Brueckner and Hawkinson (1934) found that grouping types of items on one test resulted in better achievement on a second test where types were not grouped. Capron (1933) found no difference in number of process errors on tests in which problems were arranged in random order, from easy-to-hard, or from hard-to-easy. In testing of division of decimals, Grossnickle (1944) found random sequence of items was more difficult than when items were grouped by type. The "atmosphere" of the test situation was found to be a significant factor by Goodwin (1966). The interview technique propounded by Brownell (1936) was modified by Gray (1966). Hartlein (1966) found coded items to be effective, while Graham (DA 1967) used scalogram analysis.

What types of tests are reported?

In research reports, development of the following types of tests have been reported:

(1) Readiness for division (Brueckner, 1940)

(2) Readiness for first grade arithmetic (Brueckner, 1947; Hildreth, 1935; Ferguson, DA 1967)

(3) Readiness for signed numbers (Olander, 1957)

(4) Readiness for fractions (Souder, 1943)

(5) Vocabulary (Chase, 1961)

(6) Problem solving (Connor and Hawkins, 1936)

(7) Fundamentals (Courtis, 1909, 1911; Foran and Lenaway, 1938; Olander, Van Wagenen and Bishop, 1949)

(8) Understanding (Van Brock, 1965; Ashlock, 1968; Ashlock and Welch, 1966; Flournoy, 1967a, 1967b, 1968; Hartlein, 1966)

(9) National survey tests (Romberg and Wilson, 1968)

(10) Geometry (Weaver, 1966)

(11) Arithmetic principles (Welch and Edwards, 1965)

In addition, of course, tests were developed as one aspect of many other studies.

Most of the studies in this category are only appropriate to the time in which they were done: the findings are not generalizable to today.

How do current pupils compare with pupils of the past?

No recent studies were noted.

How does "modern mathematics" achievement compare with "traditional mathematics" achievement?

S.M.S.G. pupils scored higher than traditional pupils in junior high (Cassel and Jerman, 1963), though lower scores in eighth grade were found by Williams and Shuff (1963). For other modern programs, Ruddell (1962) reported higher achievement than for traditional programs, and Payne (1965) summarized studies to conclude that modern programs are as effective as traditional programs in developing traditional mathematical skills.

How does grading affect achievement?

Students were found by Christensen (1968) to gain more when they were not graded. Dobbs and Neville (1967) found that achievement gains for a promoted group were greater than for non-promoted pupils.

How does mobility affect achievement?

Evans (1966), Perrodin and Snipes (1966), and Miller (DA 1967) found that mobility did not adversely affect achievement. Snipes (1966) reported that students from other states, who moved to Georgia, had higher achievement scores.

How does age and grade placement affect achievement?

The chronological age of a child may deter or facilitate his academic achievement and the relationship should not be overlooked in evaluating achievement progress. Though the usual procedure is to assign children to grade level by chronological age, the children in a specific grade may still represent a wide range in age.

A study by Carroll (1963) found overage third grade children scored significantly higher in arithmetic achievement, and were rated higher on attention span, independence and social maturity when compared to underage children. The findings confirm an earlier study done by Carter (1956) which found that older children (grade one through six) seemed to have an advantage over younger children in achievement. Klausmeier and others (1958) found five physical measures of organismic age contributed very little to mental, reading, language and arithmetic scores.

Several studies by Holmes and Finley (1955, 1956, 1957) dealing with 5th, 6th, 7th and 8th graders found low correlations between arithmetic achievement and grade placement deviation. Grade placement deviation was determined by the difference between children's actual grade placement and the grade they would have been placed in as defined by age.

It would appear that the effect of age on achievement may diminish, as age increases. Messler (1961) found no differences in achievement for the 8th and 9th graders having duplicate algebra courses, and concluded that age was not detrimental to achievement.

What is the relationship of intelligence to various aspects of arithmetic achievement?

The relationship between intelligence and achievement has been investigated in numerous studies. It has been an accepted fact that mental ability plays an important part in academic success. Levels of intelligence scores have developed widely used references, as superior, bright, average, normal, dull and mentally handicapped that have implicit meanings in relation to achievement. The studies investigating relationships between arithmetic achievement and intelligence indicate that the relationship does exist, but also add some qualifying dimensions. Studies verifying the fact that intelligence is highly related to total arithmetic achievement include one by Erickson (1958) with a correlation of .72 for the total sample. Studies by Rose and Rose (1961) and Gunderson and Feldt (1960) found a significant relationship, and a study by Shine (1961) found significant relationships between all but two Stanford-Binet items and arithmetic achievement. Hinkelman (1955) found school grades as an indication of achievement significantly related to intelligence.

The correlation of intelligence with sub-groupings of arithmetic achievement, in studies by Rose and Rose (1961), Capps (1962), and Erickson (1958), found the relationship to be lower in significance. Gunderson and Feldt (1960) found verbal intelligence more closely related to various areas of achievement than non-verbal intelligence, but the smallest difference was in arithmetic achievement.

Several studies have investigated variables involved in the relationship of the level of intelligence to achievement, with interesting results. Brown and Lind (1931) found children of lower intelligence generally had higher achievement in relation to their mental age. Holowinsky (1961) found students of lower ability showing better achievement in arithmetic than reading, and felt the differences in correlations between I.Q. and achievement were a function of age. Related findings from Jarvis (1964) found the actual range of achievement increased with a decrease in intelligence score.

Achievement gains in relation to intelligence were also investigated by Scott (1963) and Woodrow (1945). Scott found a higher correlation between intelligence scores and arithmetic reasoning than other subjects, and the greatest variation in arithmetic computation. Woodrow concluded that the gains seemed to result from varying combinations of factors.

Is intelligence the best indicator of expected achievement?

Though intelligence may be considered a good indicator of ability, it is not always the best indicator of achievement. Allen (1944) found achievement test scores a better predictor than intelligence scores. Arithmetic measures were found to correlate higher with reading (.49) and listening (.41) than intelligence (.23 and .21) in a study by Cleland and Toussaint (1962). Another dimension is added by a study where Furtin (1956) found general experience correlated higher than intelligence with achievement. A study by Coffing (1941) found a positive relationship between scores in paragraph meaning and arithmetic reasoning. Fay (1950) reported results that when chronological and mental ages were controlled, superior readers did not achieve more than inferior readers.

How does a child's level of anxiety affect his achievement?

The relationship of anxiety to arithmetic achievement, intelligence, and sex have been investigated by several researchers. Feldhusen and Klausmeier (1962) and McCandless and Castaneda (1956) found anxiety scores significantly related to intelligence for fifth and sixth grade girls. The first study found significantly greater correlations between anxiety and low I.Q. groups, and between anxiety and arithmetic achievement for the low group when compared to the average or high I.Q. groups. Ridding's (1967) findings add another dimension in that he found no significant relationship between anxiety and over- or under-achievement.

What effect does the mathematical knowledge of parents have on the mathematical knowledge of children?

A very important factor in a child's learning of mathematics may be the assistance he receives from his parents. To some extent, the type and amount of assistance will be related to the parents' knowledge of the subject. Three studies found that increased parent knowledge of mathematics or classroom activities resulted in higher achievement by pupils. Duncan (1964) reported that knowledge of S.M.S.G. mathematics by parents resulted in significantly higher achievement by their children, and Sitts and Sitts (1963) found that informing parents of classroom activities seemed to increase achievement. Mayes (DA 1966) found parent participation in a program resulted in higher pupil achievement. One factor that could have influenced the results of these studies is that the supportive interest of parents in their children may have been reflected or related to their willingness to gain knowledge. Another interesting dimension is added by a study by Stendler (1951). He found that generally, for pre-school children, lower-social level parents emphasized counting and higher-social level parents emphasized language as a skill needed for school.

What effect does the background of the teacher have on student achievement?

It is true that one cannot teach what one does not know. It also seems true that teachers of elementary school mathematics who have studied mathematics for some time or in great depth should be able to bring their experience to the classroom resulting in greater achievement by those students thus exposed. Bassham (1962) found that this was true. Teachers with more experience in mathematics had pupils with greater achievement in mathematics. Shim (1965) supported this finding in-so-far as the measurement of teacher experience in mathematics was not in terms of grade point average, time in college, length of certification, etc. In general, teachers with a greater understanding of mathematics were able to share that understanding with their pupils.

What effect does in-service education for teachers have on student achievement?

Studies by Houston and DeVault (1963), Ruddell and Balow (1963), Ruddell and Brown (1964), and Hurst (DA 1968) all confirm the fact that teachers involved in in-service education in elementary school mathematics are able to bring this experience to the classroom resulting in greater achievement by their pupils. Studies by Rouse (DA 1968) and Lampela (DA 1966) were in disagreement. Scaramuzzi (1956) found that teachers who are able to apply their imagination to the solution of problems in motivating elementary school mathematics also found a greater level of pupil achievement.

What kind of teaching techniques improve transfer?

The basic idea of transfer infers that something learned in one situation can be applied or used in another situation. A major concern of teachers is that pupils transfer learning from one situation to another. Two studies done in 1930 (Overman, 1930; Woody, 1930) found that emphasizing generalizations during instruction increased the amount of transfer to untaught arithmetic problems. Related to this are the results of a study by Cluley (1932), where pupils taught objectively (involving generalizations) appeared to transfer more learning than pupils who were given extra practice and/or taught by formal rules. Teaching by formal rules infers mechanical or rote instruction rather than meaningful instruction. Brownell (1949) found meaningful instruction aided transfer of learning when compared with mechanical instruction. Discovery-type instruction seems to increase transfer. Two studies (Scandura, 1964; Worthen, 1968) did find greater transfer resulted from discovery-type instruction than did from expository instruction.

How can pupil ability to transfer be increased?

The transfer of learning to new concepts and situations cannot be taken for granted by teachers. Wittrock and Keisler (1965) found specific and class cues were more effective than general cues for transfer to new situations of previously learned concepts, but transfer to new concepts was not significantly affected by specific, class, or general cues. In an experiment by Kolb (1967) mathematical instruction was specially geared for transfer to science and transfer did occur. The instructional sequence in mathematics was constructed on the basis of a mathematical hierarchy and related to quantitative science behaviors. The use of a concept name by preschool children was related to increased transfer differentiation in a study by Spiker and Terrell

(1955). It would seem that for transfer to new concepts to occur, teachers must plan and initiate the transfer.

How does age affect transfer ability of pupils?

Several studies support the idea that the age of children is related to the amount and type of transfer that can be expected of them. A study by Stevenson and Bitterman (1955) found that young children (age 4-6) could transpose to paired stimuli that were close in distance, but not to paired stimuli that were farther apart. Marshall (1966) found preschool children (age 4 1/2 to 5 1/2) were at all levels of knowledge of the middle size concept. Zeiler and Gardner (1966) found that slightly older children (age 7-8) had decreased transposition with increased differences in stimuli; verbalization did not seem to have an effect on transfer. Wohlwill (1960) found that with an increase of age (7-12 years) children used less transposition, or relational transfer, and more absolute transfer.

How much transfer of computational facts can a teacher expect?

The amount of transfer is greatest when the problems are of the same structure and transfer is to a different example of the same concept, rather than a different concept. Some older studies concerned with computational transfer found that pupils did not need to be instructed in all combinations of an operation. Knight and Setzafandt (1924) found pupils instructed in a limited set of denominators scored as well as pupils instructed in the complete set, and Olander (1931) had the same results with instruction of addition and subtraction combinations. Grossnickle (1936) found that multiplication knowledge did not transfer completely to long division, with increased errors of multiplication occurring in long division computation. It seems that a teacher can expect greater transfer of computation with similar problems, and decreasing transfer with increasing differences in the types of problems, and should plan instruction that will insure transfer to different types of problems.

What is the relationship between "meaningfulness" and retention?

A generally accepted fact is that when something being learned has meaning to the learner and is understood by the learner, the learner will be more likely to remember or retain the learning. Several studies have investigated and compared retention resulting from meaningful learning versus mechanical learning. The findings show that teaching for meaning and understanding aid retention. A study in 1949 by Brownell and Moser found this to be true as did one by Gray (1965). Shuster and Pigge (1965) found that pupils who spent 75 to 50 per cent of class time on developmental meaningful activities and less time on drill had significantly better retention than pupils who spent 25 per cent of their time on developmental and meaningful activities, with proportionatly more time on drill. Krich (1964) also found a meaningful method of teaching division of fractions aided retention.

What instructional techniques can a teacher use to produce greater retention?

Various techniques that can be used to increase retention are suggested by research, and they generally support accepted aspects of learning theory. Gagne and Bassler (1963) found that smaller variation in task examples resulted in significantly lower retention of subordinate knowledge of elementary nonmetric geometry tasks, but not of the final task. Two studies that were concerned with the retention by children of low, average and high intelligence were by Klausmeier and Check (1961) and Klausmeier and Feldhusen (1959). Both concluded that by assigning learning tasks appropriate for the achievement and intelligence level of a pupil, equal retention results for all pupils. Wittrock and Kessler (1965) found that giving specific and class cues in instruction are more effective than general cues for retention of previously learned concepts.

With re-testing, retention was found to increase in two studies by Davis and Rood (1947) and by DeWeerdt (1927). Burns (1960) concluded that intensive review as an instructional technique favors retention. The resulting retention from different methods of teaching a specific procedure were investigated in two studies. Treadway and Hollister (1963) found that teaching three cases of percentage as parts of the whole aided the average I.Q. pupils. Stephens and Dutton (1960) did not find any significant difference in retention when pupils taught division of fractions by the inversion method were compared with ones taught by a common denominator method.

What is the relationship between "discovery" type teaching and retention?

If either immediate recall or retention at a later date take precedence, different teaching methods may be appropriate. The intellectual characteristics of the pupils may also need to be considered in determining what type of instructional techniques to use. Worthen (1968) found that expository instruction resulted in higher immediate recall, but guided discovery favored retention. Meconi (1967), using programmed material, found no differences in retention for mathematically gifted pupils with different instructional techniques. These techniques included rule and example, guided discovery, and discovery.

What can teachers do to increase pupils retention of learning during the summer session?

Teachers are concerned about the lack of retention which is apparent after a summer vacation. The amount of loss of skill and achievement that occurs during summer months seems to vary with the child's ability, age, activities, and conditions of actual learning, especially when the first learning was done just prior to vacation. An older study by Osburn (1931) concluded that the greatest summer loss occurred in grades where subject-matter had been taught for the first time. Significant loss in computation and problem solving scores of fifth grade pupils seemed to be a result of use and possibly meaningful first learning in a study by Sister Josephina (1959). Scott (1967) found no systematic relationship of summer loss to the type of program, whether traditional or modern.

Two studies give teachers indications of how to decrease the amount of loss, or improve retention over the summer vacation. Dougherty (1962) found that helping children diagnose their own errors during instruction seemed to result in higher retention. Cook (1942) found that using practice materials during the summer increased retention of fundamentals for primary grade children, and the increase in retention was in direct ratio with an increase in number of weeks the practice materials were used.

What is the relationship between generalization and mathematical achievement?

Research involving generalization is scattered widely within the field of mathematical achievement. Collier (1922) studied generalization of solutions to problems involving multiplication of fractions by whole numbers. Research by Mitchell (1929) and Henderson (1967) support the general finding that given a specific task, involving specific numbers and relationships, a student can find the solution and generalize to the broader mathematical concept. Ebert (1946) found large variations in generalization ability, depending on the mathematical concept, the student's mental age or intelligence, and the visual pattern presented. Shepard (1956) also found visual patterns and geometric shapes significant in learning mathematical concepts. Overman (1930) reports that in a study of transfer, generalization produced over twenty per cent of the transfer, more than any other means. Kyte (1967) found that brighter students require a shorter time to learn fractions than less bright students.

Other questions: How can mathematics be taught so pupils develop the ability to generalize?

What generalizations have the most promise for future learning?

What is the relation of reasoning to mathematical ability?

Winch (1911) demonstrated that pupils who practiced computational problems in mathematics did better on a test of reasoning than students who practiced with problems in art, history, and English. There have been no contradictory reports. In addition, Dahle (1940) found that reasoning and long-division relate the least due to stress induced by long-division. It should be noted that a different procedure for teaching long-division is now in use, and thus these findings may not be relevant today. Wilson (1936) added the interesting report that students who were asked to correct their own papers showed less understanding of the material than those who were given practice in the correct procedures. Swineford (1949) confirmed the relationship between mathematics and reasoning in demonstrating numerical "set," the prejudice against mathematics and numbers which can influence mental behavior in other areas. Shepard (1956) pointed out that where students had learned the mathematical concept they could perform any task involving that concept. Yeager (1967) supported the relationship and limited it by showing that the rate of learning was specific to the task.

How are process and reasoning affected by rote learning in contrast to learning by discovery?

Wilson (1967) compared learning by rote and learning by discovery. He found the discovery method superior. Meconi (1967) qualified the result by showing that pupils with high ability were able to learn under any teaching method. Previously, Brownell (1943), after extensive investigation, concluded that drill does not lead to understanding. Wohlwill (1963) supported this finding and reported that in elementary school mathematics, understanding was achieved through relationships rather than memorized absolute rules. Earlier studies by Meyers (1928) and Rosse (1930) compared various forms of rote learning. Though not stated explicitly, both found achievement to be greater in situations that involved less absolute rote learning.

Is there a relationship between reasoning and chronological development?

Perreault (1957) discovered that the child's ability to count, to group, and to subitize proceeded in order, appearing as developmental stages. This led to the conclusion that reasoning in elementary school mathematics is related to developmental stages of the pupil. Brownell (1944), after extensive investigation, concluded that grade four is the earliest grade demonstrating maximum learning. Potter (1968) reported that among preschool children the ability to count was related to age more than any other factor. Harrison (1934) reported similarly that the ability to deal with the concept of time was also correlated with age and grade development. Beilin and Gillman (1967) reported in an excellent study the relationship between developmental stages and the language factor involved in numerical patterns. This study has major theoretical implications rather than practical applications.

How best can motivation in learning mathematics be increased?

There are many theories about motivation and its effect on learning. Research is not conclusive nor in agreement as to which theory is the most effective. Studies by O'Brien (1928), Brown (1932), Bouchard (1951), and Leibowitz (1966) report that knowledge of results and knowledge of the competition are the most effective means to motivation. Brown reported from the junior high level and Leibowitz from kindergarten that in controlled experiments that the pupil's knowledge of his own as well as his classmates' progress results in greater achievement.

What materials can be used to motivate elementary school mathematics?

Throughout the literature there are numerous reports about various devices and games that have been used to increase the student interest and hopefully achievement in elementary school mathematics. Scaramuzzi (1965) used money and its manipulation to teach arithmetic. Wilson (1922) presented word problems in the form of drama. Worden (1931) found games to be a better motivator of arithmetic accuracy than praise-punishment. Steinway (1918) found number games effective in the first grade. Richardson (1920) reported that setting definite goals or "Campaign Programs" increased achievement through motivation in grades four to eight. Reavis (1917) found that learning about classroom stocks and bonds motivated mathematics achievement. Goforth (1938) effectively used the game "ADD-O" to motivate greater mathematical achievement. It is obvious from all of these reports that where teachers involve their students in games or imaginative programs, the mathematical achievement of the students increases.

Is individual instruction useful in motivating mathematical achievement?

With the advent of individualized instructional media there have been several studies dealing with the motivational aspects of individualized instruction. MacLatchy (1942) reported that individualized instruction in grades three and four increased the students' motivation to achieve in elementary school mathematics. As long ago as 1915 individualized instruction has been recognized as one method to increase attitude and achievement. Anthony (1915) reports increased attention and "proper" attitude when students were given individualized instruction. The limiting factor, of course, is teacher time.

Is drill or practice helpful in motivating students?

Motivational aspects have been reported as a by-product of drill by Hoover (1921) and Hahn (1914). However, this may not be true today with today's definition of drill. Ballou (1916) reported the motivational effects of using the same achievement test every year. Wertheimer (1920) reported that motivation to achieve was increased by using a diagnostic test. All of these studies measured motivation by inference from increased mathematical achievement.

What verbal technique can teachers use to increase motivation?

Hurlock (1925) reported that praise and reproof (verbal punishment) were both able to produce an increase in motivation to achieve in elementary school mathematics, as opposed to being ignored. Worden (1931) found reproof to be more motivating than praise. However, in an excellent study, Kapos (1957) found that praise in varying amounts and in varying patterns produced excellent motivation. Hollander (DA 1968) cited evidence that verbal praise and a candy reward were more effective than no incentive or reproof.

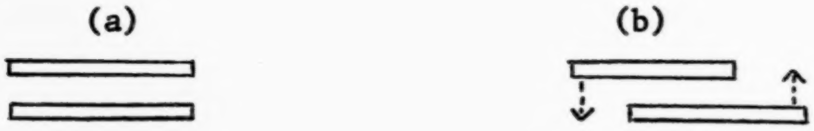
The major findings of Piagetian research related to mathematics are summarized in this section: (1) conservation, (2) transitivity, (3) perception, and (4) classification and seriation.

Conservation: Definition

The general concept of conservation refers to whether a child can maintain that an object remains the same in the face of changes in the appearance of that object. Of the studies reviewed, six types of conservation are involved. These use conservation of (1) substance, (2) length, (3) number, (4) weight, (5) distance, and (6) volume.

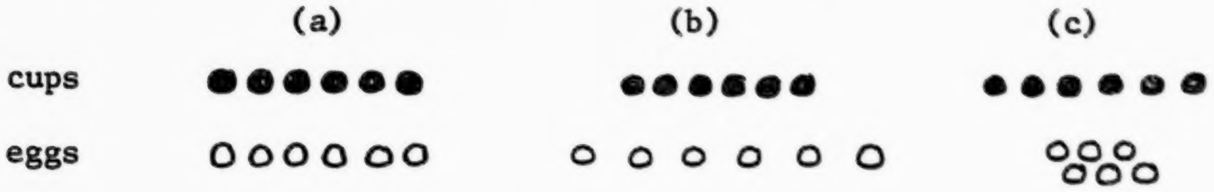
Conservation of substance frequently is termed mass or quantity. An illustrative example of a conservation of substance task is the classical plasticine (or clay) ball situation developed by Piaget. A child is shown two plasticine balls having equal amounts of clay. After he is satisfied that both balls contain the same amount of clay, the examiner takes one ball and rolls it into a hot dog shape and asks the child whether the ball and the hot dog contain the same amount of clay or whether one has more than the others. Those who are fooled by the change in shape claim that they do not have the same amount. Some children think the clay ball contains more because it "is fatter." Others may think that the hot dog contains more because it "is longer." A conserver of substance believes the two shapes to contain the same amount of clay. In order to be considered a conserver a child must be able to support his choice by giving a logical reason such as: "If you were to roll the hot dog back into a ball they would still be the same," or "It doesn't matter what shape you make the clay, they will be the same."

A typical task for conservation of length is to place two sticks of equal length parallel to one another as in (a) below. Then, one of the sticks is shifted (as in b) so that the end points are no longer lined up.



After the child is satisfied that the sticks in situation (a) are of equal length, situation (b) is created and he is asked whether the sticks are the same length or whether one is longer than the others. An additional transformation could be performed by placing one stick perpendicular to the other and asking whether the sticks were still the same length. A non-conservation response is one in which the child perceives one of the sticks to be longer than the other. If the child can retain the notion that the sticks remain the same length regardless of the change in their spatial relationship, he is considered to be a conserver.

When testing for conservation of number, a task frequently used by Piaget was the egg/container situation, in which eggs were lined up with egg cups (see a below). The child was asked to place each egg into its corresponding cup to demonstrate that there were exactly the same number of eggs as cups. The eggs were then taken out of their cups and placed in a row parallel to the cups but forming a longer row than the cups (situation b). The child was now asked whether there were the same number of eggs as cups. Another transformation which was frequently performed was bunching the eggs together (situation c), followed by a question asking whether there were the same number of eggs as cups.



If the child failed to retain the notion that the number of eggs and cups remained constant throughout changes in their spatial location, he was regarded as a non-conserver.

To illustrate conservation of weight, we can return to the plasticine ball situation described under conservation of substance. Here the child might be asked whether the two balls weighed the same. If he thought they were different, he would be invited to add or remove clay until they were the same. Then the ball would be transformed into a hot dog shape or pulled apart into three or four pieces and the child asked whether they would weigh the same. A non-conserver of weight would maintain that the clay ball and the transformed clay would no longer weigh the same, whereas a conserver of weight would maintain that the weight remains constant even though one changes the appearance of the original object.

To illustrate conservation of distance, a task used by Shantz and Smock (1966) will be described. Two 2 1/2 inch trees were placed in front of a child, about eight inches apart. The child was then asked whether the trees were far apart or near together. A board, taller than the trees, was placed halfway between them. The child was now asked the question again. They found that conservers of distance maintained that the distance remained the same regardless of whether the space was filled or empty. Non-conservers, however, saw the distance between the trees as altered. The most common non-conserver response was that the distance was less because the board used up some of the space.

Conservation of volume has been assessed by using a task such as the plasticine ball situation described in both substance and weight conservation. After the various transformations of the shape of the clay, the child is asked whether they take up the same amount of space or the same amount of room. Here again conservers of volume see the amount of space taken up as the same from transformation to transformation. Non-conservers see the volume as changing across transformations.

Order of development of conservation

Among Piaget's theoretical ideas is that of certain fixed sequences in which intellectual development occurs. One of these sequences is hypothesized to occur for the appearance of conservation, weight, and volume in the order mentioned.

Elkind (1961b) obtained results which were in close agreement with Piaget's findings for a regular, age-related order for the emergence of conservation of substance, weight, and volume. Further confirmation of this hypothesis came from the research of Uzgiris (1964).

In a study of the development of the number concept, Wohlwill (1960) concluded that the developmental process could be adequately described by three fairly discrete phases: perceptual, conceptual, and relationships. Support was thereby found for Piaget's view of a relatively uniform developmental sequence.

Coxford (1963) presented charts which summarize the work of Piaget and other investigators. The charts show the age-related order of development of various number concepts and other related concepts. Etuk (DA 1967) reported partial support for the contention that conservation, seriation and classification develop simultaneously.

Conservation: Training research (the effect of special training on the development of conservation concepts)

1. Substance. In a study which tested different methods of teaching principles of correspondence and conservation to children, Feigenbaum and Sulkin (1964) found that reduction of irrelevant stimuli was more successful than reinforcement by addition and subtraction. Children who learned the concept tended to retain it at least on a short-term basis. Gruen (1965), working with kindergarten children, found that a significant improvement in ability to conserve substance occurred for those receiving a conflict, without-verbal pretraining treatment condition.

2. Length. Murray (1968) found that non-conservers of length who were trained by a reversibility and cognitive conflict procedure did significantly better than untrained non-conservers.

3. Number. Wohlwill and Lowe (1962) studied the effects of four conditions of training on the development of non-verbal conservation. Although no significant differences were found between the training conditions, overall difference scores differed significantly from zero showing that for the total group conservation did increase. Direct training seemed no more effective than intermittent practice. Transfer of conservation learning to the verbal posttest was negligible under all conditions, indicating a rather restricted type of learning. Wallach and Sprott (1964) provided first graders with either no practice or practice in manipulating objects to develop conservation of number by reversibility. None of the no-training group achieved growth in conservation, while fourteen of fifteen trained children evidenced growth on one test, and thirteen of fifteen on another test. This effect was not diminished after 14 to 23 days.

Gruen (1965) utilized six conditions, verbal or non-verbal pretraining combined with either no training, direct training on number conservation, or "cognitive conflict" training. The conflict-plus-verbal pretraining group made significantly more number-conserving responses than children with no verbal pretraining. The conflict treatment seemed to account for much of this difference. It was also found that a subtraction/addition operation was easier and appeared earlier in the developmental sequence than the addition/subtraction operation, although some children could do both successfully and still not conserve.

Wallach, Wall, and Anderson (1967) induced children to conserve number by experiences with reversibility, while experience with addition and subtraction had no effect. The reversibility training, the authors point out, may have been successful because it led children to stop using misleading perceptual cues.

Winer (1968) attempted to test an hypothesis that practice in addition/subtraction or in evaluating length change would induce a set to respond in the practiced manner to a conflict resulting from changes in length opposing change in number. This as well as a second experiment examined the effect of this training on acquisition of conservation. Support for the effectiveness of the training procedures was found in the first experiment but not for the second.

An experiment conducted by Pace (1968) indicated that an experimental group receiving a special training program incorporating organized experiences with sets attained a higher level of number conservation than a control group who received only the regular math program. Number conservation stage placement was more closely related to I.Q. than to C.A. Implications for instruction in elementary school math are also presented.

4. Weight. Smith (1968) compared two procedures for accelerating conservation of weight in children. The methods involved were modifications of Smedslund's addition/subtraction technique and Beilin's verbal rule instructional procedure. The results indicated that Beilin's procedure produced significant improvement in conservation for both transitional conservers and non-conservers. Smedslund's procedure seemed to have little effect on either group.

Development of conservation

1. Substance. Elkind (1961b) noted that in Piaget's early studies, different tests were assigned to the age level at which 75% passed. Elkind felt that it is safe to assume that Piaget is using the same criterion when he assigned age ranges to various conservation concepts. Conservation of substance, according to Piaget, appeared in most children by ages 7-8. Elkind (1961b) found support for Piaget's age levels and noted that non-conservation explanations decreased while

conservation explanations increased with age. Uzgiris (1964) reported findings which also compared closely with the age ranges obtained by Piaget and by Elkind. Feigenbaum (1963), in an experimental study, found significant differences between performance of groups younger and older than 65 months on all treatments which differed in task complexity. The evidence suggested that the stages of development during acquisition of conservation were not defined by definite age barriers, but rather descriptive general trends. It was further noted that a child's grasp of the conservation concept tended to vary with intelligence and the nature of concrete experimental conditions. Silverman and Schneider (1968) tested for conservation of quantity without dependency upon a child's statement of "more" or "less" and found support for Piaget's age levels.

2. Length. Murray (1965) cited research findings of Piaget and his associates which indicate that conservation of length tends to appear primarily between the ages of 7 and 8. In his own study, Murray used illusion-distorted lengths and found that first graders had a significantly lower median number of conservation responses than second or third graders. Second and third graders were not significantly different. It was concluded that the transition from non-conservation to conservation occurred between ages 7 and 8, which is in support of Piaget's findings. Sawada and Nelson (1967) contended that some children may not understand precisely what the examiner means when questions are asked as to whether two sticks are the same "length" or whether one is "longer." They proceeded to develop a non-verbal means of assessing conservation of length which revealed that nearly 100% of the children between ages 7-2 and 8-0 were conservers of length. Nearly 70% of those between 6-3 and 7-1 were conservers and about 60% of those between 5-4 and 6-2 were conservers. Hence, the threshold age for conservation of length appeared to lie between ages 5 and 6 when assessment procedures follow the non-verbal technique used in this study.

Such a finding is in contradiction to the results of other work, including Piaget's. There are, of course, major procedural differences between this study and those of Piaget and others.

In a later study Murray (1968) found the transition from nonconservation to conservation of length to be between ages 6 and 7, which is contradictory to his earlier finding (Murray, 1965) in which the age range was 7 to 8. Murray's own explanation of the difference was that older children were subjects in the first study and that in the 1968 study the length conservation task might be simpler and more concrete.

3. Number. Coxford (1963) presented tables listing various number concepts in their approximate order of development and the approximate age of attainment as determined by Piaget and other researchers. He cited Piaget as indicating that most children acquired number conservation between the ages of 6 to 7 1/2.

Drawing on the ideas of Piaget and other theorists, Wohlwill (1960) found that the observed order of difficulty of seven tested tasks corresponded closely to predictions. It was concluded that the developmental process may be adequately described by three fairly discrete phases: perceptual, conceptual, and relationships. In that a relatively uniform developmental sequence was demonstrated, the theoretical views of Piaget were supported.

Nicholls (1963) demonstrated that there is a wide variability among slow learners to attain developmental concepts, such as conservation of number. He emphasized the need for pre-testing for conservation concepts before beginning courses of study.

In a methodologically weak study, Estes (1956) attempted to replicate Piaget's findings with respect to conservation of number. Using children from 4 to 6 years of age, she was unable to find reliable differences between the age groups. No evidence was obtained in support of Piaget's theories on stage development or age levels in the acquisition of mathematical and logical concepts. Procedural differences, methodological weaknesses, and the very small sample employed combine to render this study an inadequate attempt at replication.

4. Weight. Elkind (1961b) cited Piaget as assigning the age range of 9 to 10 as the period in which most children acquire conservation of weight. If it can be assumed that Piaget was using the criterion of 75% passing, then clear support was found by Elkind's replication study, in which 73% of his nine-year-old group conserved weight. Uzgiris (1964) reported the percentage of children conserving on different materials. For her fourth grade group (mean age 10-0), percentages somewhat lower than 75% were reported across the four materials used.

5. Distance. Shantz and Smock (1966) cited studies by both Piaget and independent investigators which indicate that the use of a coordinate system appeared from 6 1/2 to 12 years of age. This represents a very large age range relative to other conservation concepts. The general age at which conservation of distance emerges is, according to Piaget (as cited by Shantz and Smock), about 7 years of age, while the general age for the appearance of the coordinate system is about 9.

In order to test Piaget's hypothesis that the concept of distance conservation is a prerequisite for the concept of a coordinate system, an experiment was conducted. The differential effects of two- and three-dimension stimuli on performance were compared. In general, the data supported Piaget's hypothesis. Presentation of objects before drawings tended to facilitate more current responses than the reverse order.

6. Volume. Elkind (1961b) cited Piaget as assigning the age range of 11 to 12 as the period in which most children acquire conservation of volume. Assuming that Piaget was using the criterion of 75% passing as the basis for assigning these age levels, comparisons with the findings of other studies may be made. In Elkind's replication study, only 25% of the eleven-year-old children conserved weight, although his procedure differed from that of Piaget. No data are available on older children, since the eleven-year-olds were the oldest in the sample.

Uzgiris (1964) reported percentages for conservation of volume for the various materials used which appear quite similar to those of Elkind (1961b). Here again, an exact replication of Piaget's procedures was not carried out. It is possible, as Elkind suggested, that the task employed by Piaget was somewhat easier and, consequently, children more readily gave conservation responses. All of which suggests that procedural differences account for much of the differences from study to study, and raises some questions concerning the generality of conservation across tasks.

Generalizability of conservation concepts

In order to test the effect of varying the materials used to test for the presence of conservation of substance, weight, and volume, Uzgiris (1964) employed plasticine, metal nuts, wire coils, and straight plastic wire in her investigation. For each grade level (1 to 6) correlations were computed between the scores for conservation on each material. The results indicated that considerable consistency existed between materials although there was some degree of variation from grade to grade.

Pratoomraj and Johnson (1966) used five different tasks in testing for conservation of substance. However, other conservation concepts may have been involved with several of the problems. As a result, generality is not being tested in the same way as Uzgiris (1964) had done. They concluded that conservation of substance responses seem situation-specific at the younger ages and appear to become relatively general, and therefore, independent of the stimulus material used by age seven.

Relation of conservation to achievement

In an investigation of relationships between general intelligence, conceptual development, and school achievement in a two-year longitudinal study, Freyberg (1966) obtained results confirming previous findings that concept development is more closely linked to growth of general intellectual ability than chronological age alone. Conceptual development was measured by an objective test which assessed conservation of quantity, weight, numerical correspondence, additive composition of classes, and concepts of position in space, speed, age, kinships and causal relationships.

Using a group testing procedure with fourth graders, Overhold (1965) sought to establish whether differences in arithmetic achievement existed between conservers and non-conservers of substance. Other variables being considered were sex and intelligence. The data showed that girls achieved significantly higher intelligence test scores than boys and that conservers had significantly higher intelligence scores than non-conservers. After an adjustment for initial differences in intelligence, no significant difference in mean arithmetic achievement was found between conservers and non-conservers.

Steffe (1968) divided first grade children into four groups representing different levels of attainment of number conservation. Care was taken to ensure that the four groups had similar I.Q.'s. Those in the lowest level of number conservation performed significantly less well on a test of addition problems than the children in the upper three levels of conservation.

In a cross-cultural study, Goodnow and Bethon (1966) attempted to investigate the effect of schooling and I.Q. on Piaget's tasks by combining data from unschooled Hong Kong children and data for United States children matched on C.A. and M.A. Lack of schooling did not seem to affect conservation tasks, but did seem to affect combinational reasoning. Among school children, all tasks seemed to show a relation to mental age.

Robinson (DA 1968) reported that a child's ability to conserve, seriate, and classify is related to his level of mathematical achievement.

Transitivity: Definition

Glick and Wapner (1968) offered the following concise definition of transitivity: a transitive judgment involves the integration of two relational presentations, reducible to the form A > B and B > C, to yield the conclusion that A > C. This conclusion implies two operations (A > B > C) and the ability to reason logically on the basis of this ordinal series (p. 621).

Concrete transitivity is said to be present when a child can draw correct inferences from actual observations of real objects. Formal transitivity refers to correct inferences drawn from verbally stated, hypothetical premises. An illustration of a task involving concrete transitivity of length is to present to a child two dolls (A and B) of unequal height and ask which is the taller (or shorter). Next, the child is shown another pair of dolls (B and C), one of which is a member of the first pair. Again he is asked which is the taller (or shorter). With the information in mind that, e.g., A is taller than B and B is taller than C, the child is asked which is the tallest and/or shortest doll. A formal transitivity task that is analogous to the concrete example would be as follows: a child is told that John (A) is taller than Bill (B) and that Bill is taller than Sam (C). If, from these verbal statements, the child can correctly conclude that John (A) must be taller than Sam (C), he is said to possess formal transitivity.

Development of transitivity

Smedslund (1963b) investigated the development of concrete transitivity of length with a test designed to meet all major methodological requirements. His results supported the findings of Piaget in that the average age of acquisition of transitivity of length was about 8-0. Piaget had reported that most children acquire transitivity of length between the ages of 7 and 8. Braine (1964) criticized Smedslund's procedures in that task assignments were probably not clear to the subjects. Using a non-verbal technique he found length transitivity in most five-year-olds. Glick and Wapner (1968) used two criterion measures, (a) correctness of answer and (b) justification for answer. Both measures revealed increased transitivity reasoning with age on both verbal and concrete tasks. Adequate justifications did not always accompany correct answers, nor did inadequate reasons accompany wrong answers. Test differences were found in that more correct answers, but less adequate justificationsoccurred on the concrete test. It was noted that encoding difficulties in the verbal test introduced greater complexity than would be encountered on the concrete test.

These three studies all reflect differences in results which seem to accompany procedural variations. It thus becomes difficult to establish the stability of particular findings since they seem to be procedure-specific.

Transitivity: Training research

Smedslund (1963a) attempted to affect the development of transitivity of weight by providing different types of experiences to children between the ages of 5 and 7. Only children who practiced in ordering three objects in a series according to weight, with the help of a balance, showed definite signs of acquiring transitivity.

Perception: Size-weight illusion research

Several studies dealing with the size-weight illusion formed the basis of a report by Robinson (1964). It was found that nearly all children chose the smaller equal-in-weight object on some trials after training on choosing the heavier object rather than the bigger one. Thus they manifested the illusion to some degree. The magnitude of the size-weight illusion was found to be an inverse function of age. The younger the child, the greater the magnitude of the illusion. An experiment was performed in which children were trained to discriminate pairs of objects differing by 30 or 60 grams. It was found that the finer the discrimination, the smaller the magnitude of the size-weight illusion became. The findings were thought to contradict Piaget's specific ideas regarding the development of the illusion.

A study by Pick and Pick (1967) used the size-weight illusion as a means of investigating developmental trends in integration of the senses. Objects were presented to subjects visually, haptically (sense of touch), and both visually and haptically. The subjects included the age range 4 to 16, as well as adults. The results indicated that the developmental trends in magnitude of size-weight illusions may reflect differences in inter- and intra-modal integration, rather than age.

Perception: Logical and perceptual cues research

Halpern (1965) investigated the effects of incompatibility between perception and logic in Piaget's stage of concrete operations. Transitivity of weight tasks were presented in which both perceptual and logical cues were present. In a series of three objects (A, B, and C) in which A is heavier than B, and B is heavier than C, A could be the smallest, B the largest, with C as intermediate in size. From a logical analysis A should be heavier than C. Alternately, if a child relies on the perceptual cues he is likely to decide that C is heavier than A.

Children ranging from 5 to 7 were individually tested to determine whether they were deductively (logically) oriented or empirically (perceptually) oriented. The results indicated that children with an empirical orientation erred more often than those with a deductive orientation. Children with an empirical orientation made proportionately more of their errors when perception directly contradicted logic than did children with a deductive orientation.

Classification and seriation: Definitions

1. Comparison of quantities (see Elkind, 1961a, p. 37):
    a. Gross quantity: "single perceived relations between objects (longer than, larger than) which are not coordinated with each other."
    b. Intensive quantity: "perceived quantity relations taken two by two (longer and wider, taller and thicker)."
    c. Extensive quantity: "unit relations between objects (X is half of Y, X is twice Y, etc.)."

2. Additive composition of classes (Elkind, 1961c): The ability to additively compose classes is involved. This includes the ability to include subclasses within a total class. In Piaget's stage one, a child may be able to perceive that there were white beads and brown beads among a set of wooden beads but was unable to deal with both subclasses in a comparison with the total class. At stage two a child tends to identify a part with the whole. Hence, the brown beads may be considered as identical to the wooden beads. Finally, at the third stage a child is able to determine that the wooden beads are more than a subclass (e.g. brown beads) because there is another subclass (e.g. white beads) as well.

3. Seriation (Coxford, 1963, p. 421): Seriation refers to "placing objects in order determined by some characteristics of the objects.

For example, sticks of varying lengths can be placed in a series from longest to shortest or the reverse."

Development of classification and seriation

Elkind (1961a) found that children's success in comparing quantity increased significantly with age. The type of quantity significantly affected children's comparisons, with gross quantities easiest, then intensive, followed by extensive as hardest, thereby supporting Piaget's theory. Support was also found for Piaget's notion that a common conceptualizing ability underlies children's successes in comparing quantities with different materials.

Working with children between the ages of 3 and 5, Estes and Combs (1966) investigated the development of perception of quantity relative to the understanding of the concept "more" by using two- and three-dimensional figures. The results indicated that the concept seemed to occur between the ages of 3 and 4 for both sexes, and was slightly affected by the type and number of stimuli.

In his replication study on the development of additive composition of class concepts, Elkind (1961c) obtained results agreeing with Piaget's finding of three age-related stages in the development of ability to form class inclusions.

Another replication study was performed by Elkind (1964) in which discrimination, seriation, and numeration of size were investigated following Piagetian procedures. The results showed a regular increase with age in a child's ability. Dimensionality of materials affected ease of success, but not sequence of success, on discrimination, seriation, and numeration tasks. These results were in agreement with Piaget's.

Benson (DA 1967) reported support for Piaget's contention that the development of class and seriation are required for number conceptualization.

Answers from Research: Piagetian concepts

Coxford (1964) found developmental stage to be most probably related to age. In general, Piaget's predictionsof CA $\leq$ 57 months for stage one (no understanding of seriation) were corroborated with bright children as subjects, though exceptions were noted.

Classification and seriation: Training research

An attempt to influence stage placement on seriation tasks through instruction was made by Coxford (1964). Differences between instructed and non-instructed groups who were at stage one (no understanding) were not significant. Differences between the treatment groups who were at the transitional stage were significant. The instructed group advanced to the complete attainment stage. Age was not the sole factor, as Piaget suggests.

How effective is reinforcement for increasing the learning of mathematics?

The use of reinforcement in the learning situation is an accepted teaching technique. The methods or kinds of reinforcement and the time of reinforcement can be varied in a multitude of ways. A group of well-done experiments support the idea that reinforcement can and does increase learning, and gives clues to the classroom teacher as how and when to use reinforcement (Bouchard, 1951; Brown, 1932; Doherty and Wunderlich, 1968; Paige, 1966). A related study, done by Feigenbaum and Sulkin (1964), found the reduction of irrelevant stimuli more successful than reinforcement. It would seem that by using both reinforcement and reduction of irrelevant stimuli, learning could be increased.

What type of reinforcement seems more effective and when should it be used?

One apparent and feasible way of using reinforcement to improve learning is supported by three reputable studies. By giving information on the results of tests, Bouchard (1951), Brown (1932), and Paige (1966) all found significant gains in achievement. Brown also found that boys appeared to be more easily influenced by this type of reinforcement than girls. Varying the amount of reinforcement, rather than using a constant amount, was found to be more effective for having young children change their estimations of size (Tajfel and Winter, 1963).

Most teachers have at times prompted students by giving the correct answer rather than waiting for the student to respond. McNeil (1965) found that waiting until the student had overtly responded before giving the correct answer as reinforcement, significantly increased achievement, was even more effective in grade three than grade five, and seemed to be especially effective with low mental ability children. Doherty and Wunderlich (1968) found that increasing the amount of secondary reinforcement (an object or symbol that in itself has no immediate value, but has been paired with a primary reinforcer that does have immediate value) aided in increasing the number of problem solving tasks performed by seventh and eighth grade boys.

How mathematically competent are pre-service teachers?

The majority of the studies in this category were surveys, and reflected surprisingly similar conclusions over a period of years.

The mathematical competency of pre-service teachers:

(1) is inadequate (Creswell, 1964; Fulkerson, 1960; Glennon, 1949; Reys, 1968; Skypek, 1965; Smith, 1963; Taylor, 1938; Weaver, 1956; Callahan, DA 1967);

(2) seems to be improving (Clark, 1955; Combs, 1963; C.U.P.M., 1967; Fisher, 1967; Grossnickle, 1962; Rappaport, 1958; Jensen, DA 1967; Withnell, DA 1968).

What are effective procedures for pre-service preparation?

Mathematics courses and methods courses resulted in increased understanding of concepts and attitudes reflecting a growing appreciation of arithmetic (Dutton, 1961, 1965, 1966; Dutton and Cheney, 1964; Smith, 1967; Weaver, 1956). Strong indication of which type of course is best is lacking, though separate methods and content courses (Wickes, DA 1968), a combined content-methods course (Phillips, 1968), a CAI course (Riedesel and Suydam, 1967), and a remedial course (Dutton, 1966; Waggoner, 1958) were shown to be effective. Gibbons (DA 1968) reported that discussion classes were more effective than those without discussion, while Northey (DA 1967) could not find any proportion of time for lecture or discussion was better than any other. Use of enrichment problems was helpful (Litwiller, DA 1968). Bassler (DA 1966) used exercises which were either purely mathematical or framed in a physical world setting, but found no resulting difference in achievement.

What are the attitudes of pre-service teachers?

Attitudes of pre-service teachers toward mathematics were:

(1) majority, unfavorable (Dutton, 1951; Smith, 1964);

(2) slightly more favorable in 1962 than in 1954 (Dutton, 1962); slightly more favorable after mathematics preparatory courses (Reys and Delon, 1968; Gee, DA 1966);

(3) favorable (Kane, 1968).

Unfavorable attitudes were related to lack of understanding, disassociation from life, boring aspects, insecurity and fear of making mistakes, and difficulty (Dutton, 1951, 1954, 1962).

Favorable attitudes were related to enjoyment, importance, challenge, and good teachers (Dutton, 1951, 1954).

What is the most effective way to conduct in-service courses?

Brown (1965) evaluated one approach using consultants and workshops, and found it increased understanding and use of new techniques. Creswell (1967) found that workshops did not appear to be sufficiently effective, but Whitman (1966) reported increased scores in conceptual knowledge. Dutton and Hammond (1966) found a course using a college professor and a textbook less effective than one using district staff and a variety of instructional materials. They suggested that the second program was less structured, but more adapted to individual needs. Classroom consultant services apparently were useful (DeVault, Houston, and Boyd, 1963). Harper (1964) reported increased achievement, and Todd (1966) reported increases in both achievement and favorable attitude after a "mathematics for teachers" course. Weaver (1966) found teachers who had been exposed to geometry scored higher on a geometry inventory. Beers (DA 1968) found discussion alone to be more effective than when combined with supervised study, and Foley (DA 1966) found teachers achieved as much in a large class as in smaller classes with discussion. Correspondence courses using television and programmed materials were effective, according to Green (DA 1968). Lindsay (DA 1966) found both lecture-discussion and programmed courses were effective. Kennedy and Alves (1964) surveyed teachers, and found wide variability in their suggestions. The most agreement was expressed for courses which combined content and methods.

How do teachers feel about teaching mathematics?

Brown (1965) noted that, while teachers feel inadequate in teaching mathematics, they still liked to teach it. Bean (1959) found that teachers did not perceive themselves as competent after taking a mathematical understanding test as they had before it. Barnes, Cruickshank, and Foster (1960) reported that teachers who were judged superior tended to underrate themselves, while those judged fair tended to overrate themselves and had a more negative attitude toward mathematics. Turner and others (1963) reported several studies with the Mathematics Teaching Tasks Test, on which high scores were found to be related to high pupil achievement.

Hollingsworth, Lacey, and Shannon (1930) reported that teachers at that time thought arithmetic and reading were the easiest subjects to teach, because of (1) personal liking, (2) thorough knowledge and training, and (3) adequate texts and organized courses. It would be interesting to see a replication of this study today. Huettig and Newell (1966) reported that teachers with more than ten years of experience were less positive toward a modern mathematics program, while positive statements increased with the amount of training.

How competent are teachers to teach mathematics?

Teachers were found to be weakest in whole number, decimal, and percentage concepts (Kenney, 1965). Few processes, concepts, and relationships were understood by the majority of teachers (Orleans and Wandt, 1953; Robinson, 1935). LeBaron (1949) reported that only half of teachers' responses expressed agreement with research findings.

Stoneking and Welch (1961) reported that amount of preparation was reflected in higher scores more than age or teaching experience were, but Hand (DA 1967) found experience was a significant factor. Buck (DA 1968) failed to observe differences in teaching behaviors due to mathematics achievement or classroom experience, nor did Dickens (DA 1966) observe changes after a course, despite increased achievement.

Griffin (DA 1967) surveyed over 1,000 teachers and found that they understood only half of the total topics and one-third of the modern topics. Williams (DA 1966) also reported low levels of achievement when compared with pupils, and Kipps (1968) cited details, resulting from an inventory, of what teachers understand about mathematics.

List of Applicable, Generalized Findings

This list includes the findings which the authors believe to be clearly substantiated by research on elementary school mathematics. The items are not connected to any one study, but are generalizations drawn from many. They seem applicable to the modern curriculum, often across a wide range of grades; sometimes they may also be applicable to subjects other than mathematics.

A systematically planned program of instruction in arithmetic is better than incidental instruction.

Instruction in arithmetic should be based on the readiness of pupils.

The type of classroom organization (departmentalized, team teaching, self-contained, etc.) apparently does not affect achievement.

The teacher and the strategies he uses are important.

The teaching methods which are used can decrease the difficulty of the learning task.

Meaningful teaching is better than mechanical, rote teaching.

Meaningful teaching increases retention, transfer, and understanding.

Modern mathematics programs tend to produce better reasoning and retention, but do not improve computational skills.

Inductive discovery strategies are effective, especially for retention and transfer.

Teaching for transfer is necessary.

Transfer is greatest when content is similar.

New concepts introduced at the end of the school year are less apt to be retained over the summer vacation.

Grouping is desirable, especially within a class.

Individualizing instruction improves immediate achievement, retention, and transfer.

Motivation is important for arithmetic achievement.

Use of mathematical games increases motivation.

Elementary school pupils generally like mathematics, as do teachers.

Pupil attitude toward mathematics is related to intelligence and achievement.

Pupils have more positive attitudes toward arithmetic when it is taught as a useful skill, with practical values for out-of-school situations.

Counting money and telling time are the most frequent out-of-school uses of arithmetic skills by pupils.

Rate of learning is related to intelligence.

Intelligence is related to achievement.

Drill and practice are necessary for computational accuracy.

At least one-half of class time should be spent on developmental activities.

Drill should only be used after effective developmental activities.

Concrete materials should be used before proceeding to abstractions.

Drill should be spaced and varied in type and amount.

Periodic review increases retention.

Immediate review of arithmetic test items increases achievement and retention.

Reinforcement increases achievement in mathematics.

Verbal praise aids motivation and achievement.

Practice in mental computation should be provided.

Children know a great deal of mathematics before they enter kindergarten.

Developmental stages (such as Piaget's) appear to be related to mathematical achievement.

Many children can count by ones to ten and beyond upon entering kindergarten.

Proficiency in counting facilitates the learning of addition.

Computational errors with basic facts are the greatest source of pupil difficulty, with lack of understanding second.

Diagnosis of pupil errors can be done effectively by listening to pupils verbalize while working.

In all four operations, basic facts vary in difficulty.

The decomposition method of subtraction may be better than the equal additions method for developing understanding.

For legible numeral writing, continuous emphasis is necessary.

A variety of problem solving procedures should be systematically taught.

Specific training in mathematical vocabulary increases problem solving ability.

Problems of interest to pupils promote greater achievement in problem solving.

Characteristics of good problem solvers include higher intelligence, strong computational skills, ability to estimate and analyze, skill in noting irrelevant detail, and understanding of concepts.

Current textbooks vary widely in scope and sequence.

Arithmetic achievement is related to reading ability.

The reading level of many arithmetic textbooks is too difficult.

Programmed instruction can be used to present many topics effectively.

Socioeconomic level affects background and achievement, but not as much in mathematics as in other curricular areas.

Increased parent knowledge of classroom mathematics activities results in higher pupil mathematical achievement.

Teacher background is related to pupil achievement.

The mathematical competency of teachers is inadequate but seems to be improving.

APPENDIX A

BIBLIOGRAPHY OF RESEARCH REPORTS ON ELEMENTARY SCHOOL MATHEMATICS, 1900-1968

Abrego, Mildred Brown. Children's Attitudes Toward Arithmetic. *Arith. Teach.* 13: 206-208; Mar. 1966.

Aftreth, Orville B. A Brief Discussion of One of the Analyses in the Experiment: The Effect of the Systematic Analysis of Errors on Achievement in the Study of Fractions. *J. Exp. Ed.* 24: 273-281; June 1956.

Aftreth, Orville B. Shall We Expose Our Pupils to Errors? *Arith. Teach.* 4: 129-131; Apr. 1957.

Aftreth, Orville B. The Effect of the Systematic Analysis of Errors in the Study of Fractions at the Sixth Grade Level. *J. Ed. Res.* 52: 31-34; Sept. 1958.

Aftreth, Orville B. and MacEachern, Donald G. An Action Research Study in Arithmetic. *Arith. Teach.* 11: 30-32; Jan. 1964.

Alexander, A. M. Teacher Judgment of Pupil Intelligence and Achievement Is Not Enough. *El. Sch. J.* 53: 396-401; Mar. 1953.

Alexander, Vincent E. Seventh Graders' Ability to Solve Problems. *Sch. Sci. Math.* 60: 603-606; Nov. 1960.

Alexander, Vincent E. Sex Differences in Seventh Grade Problem Solving. *Sch. Sci. Math.* 62: 47-50; Jan. 1962.

Allen, Mildred M. Relationship Between Kuhlmann-Anderson Intelligence Tests and Academic Achievement in Grade IV. *J. Ed. Psychol.* 35: 229-239; Apr. 1944.

Allen, Robert W. The Fourth "R." *Calif. J. Ed. Res.* 16: 75-79; Mar. 1965.

Ames, Louise Bates. The Development of the Sense of Time in the Young Child. *J. Genet. Psychol.* 68: 97-125; 1946.

Anderson, George R. Visual-Tactual Devices and Their Efficacy. *Arith. Teach.* 4: 196-201; Nov. 1957.

Anderson, George R. These Eighth Graders Don't Hate Arithmetic. *Math. Teach.* 51: 267-269; Apr. 1958.

Andreen, Earl W. A Study of Workbooks in Arithmetic. *J. Ed. Res.* 32: 108-122; Oct. 1938.

Anthony, Kate, *et al*. The Development of Proper Attitudes Towards School Work. *Sch. and Soc.* 2: 926-934; Dec. 25, 1915.

Ashlock, Robert B. A Test of Understanding for the Primary Grades. *Arith. Teach.* 15: 438-441; May 1968.

Ashlock, Robert B. and Welch, Ronald C. A Test of Understandings of Selected Properties of a Number System. *Ind. U. Sch. Ed. Bull.* 42: 1-74; Mar. 1966.

Ashlock, Robert B. and Welch, Ronald C. A Test of Understandings of Selected Properties of a Number System: Primary Form. *Ind. U. Sch. Ed. Bull.* 42: 1-71; Mar. 1966.

Baldauf, Robert J. A Comparison of the Extent of Educational Growth of Mentally Advanced Pupils in the Cedar Rapids Experiment. *J. Ed. Res.* 52: 181-183; Jan. 1959.

Ballenger, H. L. Overcoming Some Addition Difficulties. *J. Ed. Res.* 13: 111-117; Feb. 1926.

Ballou, F. W. Improving Instruction Through Educational Measurement. *Ed. Adm. and Sup.* 2: 354-367; June 1916.

Balow, Bruce and Curtin, James. Ability Grouping of Bright Pupils. *El. Sch. J.* 66: 321-326; Mar. 1966.

Balow, Irving H. Reading and Computation Ability as Determinants of Problem Solving. *Arith. Teach.* 11: 18-22; Jan. 1964.

Balow, Irving. The Effects of "Homogeneous" Grouping in Seventh-Grade Arithmetic. *Arith. Teach.* 11: 186-191; Mar. 1964.

Balow, Irving H. and Ruddell, Arden K. The Effects of Three Types of Grouping on Achievement. *Calif. J. Ed. Res.* 14: 108-117; May 1963.

Banghart, Frank W.; McLaulin, John C.; Wesson, James B.; and Pikaart, Leonard. An Experimental Study of Programmed Versus Traditional Elementary School Mathematics. *Arith. Teach.* 10: 199-204; Apr. 1963.

Barnes, Kenneth; Cruickshank, Raymond; and Foster, James. Selected Educational and Experience Factors and Arithmetic Teaching. *Arith. Teach.* 7: 418-420, 430; Dec. 1960.

Barthelmess, Harriet M. and Boyer, Philip A. An Evaluation of Ability Grouping. *J. Ed. Res.* 26: 284-294; Dec. 1932.

Bassham, Harrell. Teacher Understanding and Pupil Efficiency in Mathematics - A Study of Relationship. *Arith. Teach.* 9: 383-387; Nov. 1962.

Bassham, Harrell; Murphy, Michael; and Murphy, Katherine. Attitude and Achievement in Arithmetic. *Arith. Teach.* 11: 66-72; Feb. 1964.

Bassler, Otto C. Intermediate Versus Maximal Guidance - A Pilot Study. *Arith. Teach.* 15: 357-362; Apr. 1968.

Beall, Ross H. What Is Good Fraction Drill? *El. Eng. R.* 6: 49-52; Feb. 1929.

Bean, John E. Arithmetical Understandings of Elementary-School Teachers. *El. Sch. J.* 59: 447-450; May 1959.

Beatty, Willard W. The Additive Versus the Borrowing Method of Subtraction. *El. Sch. J.* 21: 198-200; Nov. 1920.

Beechy, Atlee. Pupil Failure in the Elementary Schools. *Ed. Res. J.* 22: 99-101; Apr. 14, 1943.

Beilin, Harry and Gillman, Irene S. Number Language and Number Reversal Learning. *J. Exp. Child Psychol.* 5: 263-277; June 1967.

Bemis, Eaton O. and Trow, William Clark. Remedial Arithmetic After Two Years. *J. Ed. Res.* 35: 443-452; Feb. 1942.

Benezet, Louis P. The Story of an Experiment. *NEA J.* 24: 241-244, 301-303; Nov., Dec. 1935; 25: 7-8; Jan. 1936.

Bergen, Patricia M. Action Research on Division of Fractions. *Arith. Teach.* 13: 293-295; Apr. 1966.

Berglund-Gray, Gunborg. Difficulty of the Arithmetic Processes. *El. Sch. J.* 40: 198-203; Nov. 1939.

Berglund-Gray, Gunborg and Young, Robert V. The Effect of Process Sequence on the Interpretation of Two-Step Problems in Arithmetic. *J. Ed. Res.* 34: 21-29; Sept. 1940.

Bernstein, Allen. A Study of Remedial Arithmetic Conducted With Ninth Grade Students. *Sch. Sci. Math.* 56: 25-31; Jan. 1956.

Bernstein, Allen. A Study of Remedial Arithmetic Conducted With Ninth Grade Students. *Sch. Sci. Math.* 56: 429-437; June 1956.

Bernstein, Allen. Library Research - A Study in Remedial Arithmetic. *Sch. Sci. Math.* 59: 185-195; Mar. 1959.

Betts, Emmett A. A Preliminary Investigation of the Value of a Calculating Machine for Arithmetic Instruction. *Ed.* 58: 229-235; Dec. 1937.

Bigelow, Elizabeth B. A Study Made From the Results of the Stanford Achievement Test in Grade Two. *Ed. Meth.* 12: 232-235; Jan. 1933.

Bjonerud, Corwin E. Arithmetic Concepts Possessed by the Preschool Child. *Arith. Teach.* 7: 347-350; Nov. 1960.

Blackman, Leonard S. and Capobianco, Rudolph J. An Evaluation of Programmed Instruction with the Mentally Retarded Using Teaching Machines. *Am. J. Ment. Def.* 70: 262-269; Sept. 1965.

Blackwell, A. M. A Comparative Investigation Into the Factors Involved in Mathematical Ability of Boys and Girls. *Br. J. Ed. Psychol.* 10: 143-153; June 1940 (Part I); 10: 212-222; Nov. 1940 (Part II).

Blum, Abraham. The Value Factor in Children's Size Perception. *Child Develop.* 28: 3-14; Mar. 1957.

Bobbitt, Franklin. The Technique of Curriculum-Making in Arithmetic. *El. Sch. J.* 25: 127-143; Oct. 1924.

Bogut, Thomas L. A Comparison of Achievement in Arithmetic in England, California, and St. Paul. *Arith. Teach.* 6: 87-94; Mar. 1959.

Boss, Mabel E. Arithmetic, Then and Now. *Sch. and Soc.* 51: 391-392; Mar. 23, 1940.

Bouchard, John B. An Exploratory Investigation of the Effect of Certain Selected Factors Upon Performance of Sixth Grade Children in Arithmetic. *J. Exp. Ed.* 20: 105-112; Sept. 1951.

Bowman, Herbert Lloyd. The Relation of Reported Preference to Performance in Problem Solving. *U. Mo. Bull.* 30: 1-52; Sept. 21, 1929.

Bowman, Herbert Lloyd. The Relation of Reported Preference to Performance in Problem Solving. *J. Ed. Psychol.* 23: 266-276; Apr. 1932.

Bowman, Herbert Lloyd. Reported Preference and Performance in Problem Solving According to Intelligence Groups. *J. Ed. Res.* 25: 295-299; Apr./May 1932.

Brace, Alec and Nelson, L. Doyal. The Preschool Child's Concept of Number. *Arith. Teach.* 12: 126-133; Feb. 1965.

Bradley, R. C. and Earp, N. Wesley. The Effective Teaching of Roman Numerals in Modern Mathematics Classes. *Sch. Sci. Math.* 66: 415-420; May 1966.

Braine, Martin D. S. Development of a Grasp of Transitivity of Length: A Reply to Smedslund. *Child Develop.* 35: 799-810; Sept. 1964.

Bramhall, Edwin W. An Experimental Study of Two Types of Arithmetic Problems. *J. Exp. Ed.* 8: 36-38; Sept. 1939.

Braverman, Benjamin. Does a Year's Exposure to Algebra Improve a Pupil's Ability in Arithmetic? *Math. Teach.* 32: 301-312; Nov. 1939.

Breed, Frederick S. and Ralston, Alice L. The Direct and Indirect Methods of Teaching the Addition Combinations. *El. Sch. J.* 37: 283-294; Dec. 1936.

Brewer, Emery. A Survey of Arithmetic Intraclass Grouping Practices. *Arith. Teach.* 13: 310-314; Apr. 1966.

Brinkman, Erwin H. Programed Instruction as a Technique for Improving Spatial Visualization. *J. Appl. Psychol.* 50: 179-184; Apr. 1966.

Brooks, Jane, *et al.* Choosing a Textbook in Arithmetic. *Ed. Meth.* 16: 200-202; Jan. 1937.

Brooks, S. S. A Study of the Technical and Semi-Technical Vocabulary of Arithmetic. *Ed. Res. B.* 5: 219-222; May 26, 1926.

Brotherton, D. A.; Read, J. M.; and Pratt, K. C. Indeterminate Number Concepts: II. Application by Children to Determinate Number Groups. *J. Genet. Psychol.* 73: 209-236; 1948.

Brown, Andrew W. and Lind, Christine. School Achievement in Relation to Mental Age - A Comparative Study. *J. Ed. Psychol.* 22: 561-576; Nov. 1931.

Brown, Francis J. Knowledge of Results as an Incentive in School Room Practice. *J. Ed. Psychol.* 23: 532-552; Oct. 1932.

Brown, Gerald W. An In-Service Program in Arithmetic. *El. Sch. J.* 66: 75-80; Nov. 1965.

Brown, J. C. An Investigation on the Value of Drill Work in the Fundamental Operations of Arithmetic. *J. Ed. Psychol.* 2: 81-88; Feb. 1911.

Brown, J. C. An Investigation on the Value of Drill Work in the Fundamental Operations of Arithmetic. I. *J. Ed. Psychol.* 3: 485-492; Nov. 1912. II. 3: 561-570; Dec. 1912.

Brownell, William A. The Development of Children's Number Ideas in the Primary Grades. *Suppl. Ed. Monog.* 35: 1-241; Aug. 1928.

Brownell, W. A. Remedial Cases in Arithmetic. *Peabody J. Ed.* 7: 100-107; Sept. 1929.

Brownell, William A. The Effect of Unfamiliar Settings on Problem Solving. *Duke U. Studies in Ed.* 1: 1-86; 1931.

Brownell, William A. An Evaluation of an Arithmetic Crutch. *J. Exp. Ed.* 2: 5-34; Sept. 1933.

Brownell, William A. Readiness and the Arithmetic Curriculum. *El. Sch. J.* 38: 344-354; Jan. 1938.

Brownell, William A. Borrowing in Subtraction. *J. Ed. Res.* 33: 415-424; Feb. 1940.

Brownell, William A. Arithmetic in Grades I and II. *Duke U. Studies in Ed.* 6: 1-175; 1941.

Brownell, William A. Rate, Accuracy, and Process in Learning. *J. Ed. Psychol.* 35: 321-337; Sept. 1944.

Brownell, William A. An Experiment on "Borrowing" in Third-Grade Arithmetic. *J. Ed. Res.* 41: 161-171; Nov. 1947.

Brownell, William A. Arithmetical Readiness as a Practical Classroom Concept. *El. Sch. J.* 52: 15-22; Sept. 1951.

Brownell, William A. The Effects of Practicing a Complex Arithmetical Skill Upon Proficiency in Its Constituent Skills. *J. Ed. Psychol.* 44: 65-81; Feb. 1953.

Brownell, William A. Observations of Instruction in Lower Grade Arithmetic in English and Scottish Schools. *Arith. Teach.* 7: 165-177; Apr. 1960.

Brownell, William A. Arithmetical Abstractions - Progress Toward Maturity of Concepts Under Differing Programs of Instruction. *Arith. Teach.* 10: 322-329; Oct. 1963.

Brownell, William A. Conceptual Maturity in Arithmetic Under Differing Systems of Instruction. *El. Sch. J.* 151-163; Dec. 1968.

Brownell, William and Carper, Doris V. Learning the Multiplication Combinations. *Duke U. Studies in Ed.* 7: 1-177; 1943.

Brownell, William A. and Chazal, Charlotte B. The Effects of Premature Drill in Third-Grade Arithmetic. *J. Ed. Res.* 29: 17-28; Sept. 1935.

Brownell, William A.; Kuehner, Kenneth G.; and Rein, William C. Learning as Reorganization. *Duke U. Press* 3: 1-74; 1939.

Brownell, William A. and Moser, Harold E. Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction. *Duke U. Studies in Ed.* 8: 1-207; 1949.

Brownell, William A. and Watson, Brantley. The Comparative Worth of Two Diagnostic Techniques in Arithmetic. *J. Ed. Res.* 29: 664-676; May 1936.

Brueckner, L. J. A Technique for Analyzing the Distribution of Drill in Fractions. *Ed. Meth.* 7: 352-358; May 1928.

Brueckner, Leo J. Analysis of Errors in Fractions. *El. Sch. J.* 28: 760-770; June 1928.

Brueckner, Leo J. Analysis of Difficulties in Decimals. *El. Sch. J.* 29: 32-41; Sept. 1928.

Brueckner, L. J. A Diagnostic Chart for Determining the Supervisory Needs of Teachers of Arithmetic. *El. Sch. J.* 30: 96-103; Oct. 1929.

Brueckner, Leo J. The Development of Readiness Tests in Arithmetic. *J. Ed. Res.* 34: 15-20; Sept. 1940.

Brueckner, Leo J. The Development and Validation of an Arithmetic Readiness Test. *J. Ed. Res.* 40: 496-502; Mar. 1947.

Brueckner, Leo J. and Elwell, Mary. Reliability of Diagnosis of Error in Multiplication of Fractions. *J. Ed. Res.* 26: 175-185; Nov. 1932.

Brueckner, Leo J. and Hawkinson, Mabel J. The Optimum Order of Arrangement of Items in a Diagnostic Test. El. Sch. J. 34: 351-357; Jan. 1934.

Brueckner, Leo J. and Irving, James A. A Technique for Comparing the Difficulty of Problems in Textbooks in Arithmetic. El. Sch. J. 33: 283-285; Dec. 1932.

Brueckner, Leo J. and Melbye, Harvey O. Relative Difficulty of Types of Examples in Division With Two-Figure Divisors. J. Ed. Res. 33: 401-414; Feb. 1940.

Buckingham, B. R. Teaching Addition and Subtraction Facts Together or Separately. Ed. Res. B. 6: 228-229, 240-242; May 25, 1927.

Buckingham, B. R. The Additive Versus the Take-Away Method of Teaching the Subtraction Facts. Ed. Res. B. 6: 265-269; Sept. 28, 1927.

Buckingham, B. R. Upward Versus Downward Addition. J. Ed. Res. 16: 315-322; Dec. 1927.

Buckingham, B. R. How Much Number Do Children Know? Ed. Res. B. 8: 279-284; Sept. 11, 1929.

Buell, Robert R. Mathematics Education Research in Sweden. Sch. Sci. Math. 63: 241-244; Mar. 1963.

Burch, Robert L. Formal Analysis as a Problem-Solving Procedure. J. Ed. 136: 44-47, 64; Nov. 1953.

Burge, Lofton V. Types of Errors and Questionable Habits of Work in Multiplication. El. Sch. J. 33: 185-194; Nov. 1932.

Burge, Lofton V. The Interview Technique as a Means of Diagnosing. J. Ed. Res. 27: 422-429; Feb. 1934.

Burke, Norris F. and Anderson, Kenneth E. A Comparative Study of 1939 and 1950 Achievement Test Results in the Hawthorne Elementary School in Ottawa, Kansas. J. Ed. Res. 47: 19-33; Sept. 1953.

Burns, Paul C. Intensive Review as a Procedure in Teaching Arithmetic. El. Sch. J. 60: 205-211; Jan. 1960.

Burns, Paul C. Arithmetic Books for Elementary Schools. Arith. Teach. 7: 147-149; Mar. 1960.

Burns, Paul C. The Exploratory Use of the Three Meanings of 'The Average.' Sch. Sci. Math. 63: 332-336; Apr. 1963.

Burns, Paul C. and Yonally, James L. Does the Order of Presentation of Numerical Data in Multi-Steps Affect Their Difficulty? *Sch. Sci. Math.* 64: 267-270; Apr. 1964.

Burton, Cassie B. Results of Definite Drill in Four Fundamental Processes as Shown by the Woody-McCall Mixed Fundamentals. *Yrbk. of Dept. El. Sch. Prin.* 5: 323-328; 1925.

Bush, A. S. A Classroom Experiment in Teaching Elementary Arithmetic. *Ed. Meth.* 11: 284-289; Feb. 1932.

Buswell, G. T. Diagnostic Studies in Arithmetic. *Suppl. Ed. Monog.* 30: 1-212; July 1926.

Buswell, G. T. The Growth of Concepts of Technical Terms in Arithmetic. *Ind. U. Sch. Ed. Bull.* 6: 26-29; Apr. 1930.

Buswell, G. T. A Comparison of Achievement in Arithmetic in England and Central California. *Arith. Teach.* 5: 1-9; Feb. 1958.

Buswell, G. T. and John, Lenore. The Vocabulary of Arithmetic. *Suppl. Ed. Monog.* 38: 1-146; 1931.

Buswell, Margaret M. The Relationship Between the School Structure of the Classroom and the Academic Success of the Pupils. *J. Exp. Ed.* 22: 37-52; Sept. 1953.

Callahan, John J. and Jacobson, Ruth S. An Experiment With Retarded Children and Cuisenaire Rods. *Arith. Teach.* 14: 10-13; Jan. 1967.

Callahan, Leroy. Remedial Work With Underachieving Children. *Arith. Teach.* 9: 138-140; Mar. 1962.

Capps, Lelon R. Division of Fractions. *Arith. Teach.* 9: 10-16; Jan. 1962.

Capps, Lelon. A Comparison of Superior Achievers and Underachievers in Arithmetic. *El. Sch. J.* 63: 141-145; Dec. 1962.

Capps, Lelon R. A Comparison of the Common Denominator and Inversion Method of Teaching Division of Fractions. *J. Ed. Res.* 56: 516-522; July/Aug. 1963.

Capron, Virginia Lee. The Relative Effect of Three Orders of Arrangement of Items Upon Pupils' Scores in Certain Arithmetic and Spelling Tests. *J. Ed. Psychol.* 24: 687-695; Dec. 1933.

Carper, Doris. Seeing Numbers as Groups in Primary-Grade Arithmetic. *El. Sch. J.* 43: 166-170; Nov. 1942.

Carroll, Marian L. Academic Achievement and Adjustment of Underage and Overage Third Graders. *J. Ed. Res.* 56: 415-419; Apr. 1963.

Carson, T. E. and Wheeler, L. R. Rehabilitation in Arithmetic With College Freshmen. *Peabody J. Ed.* 8: 24-27; July 1930.

Carter, Lowell Burney. The Effect of Early School Entrance on the Scholastic Achievement of Elementary School Children in the Austin Public Schools. *J. Ed. Res.* 50: 91-103; Oct. 1956.

Cassel, Russell N. and Jerman, Max. A Preliminary Evaluation of S.M.S.G. Instruction in Arithmetic and Algebra for 7th, 8th, and 9th Grade Pupils. *Calif. J. Ed. Res.* 14: 202-207; Nov. 1963.

Cawley, John F. and Goodman, John O. Interrelationships Among Mental Abilities, Reading, Language Arts, and Arithmetic With the Mentally Handicapped. *Arith. Teach.* 15: 631-636; Nov. 1968.

Chapman, J. C. and Feder, R. B. The Effect of External Incentives on Improvement. *J. Ed. Psychol.* 8: 469-474; Oct. 1917.

Chase, Clinton I. The Position of Certain Variables in the Prediction of Problem-Solving in Arithmetic. *J. Ed. Res.* 54: 9-14; Sept. 1960.

Chase, Clinton I. Formal Analysis as a Diagnostic Technique in Arithmetic. *El. Sch. J.* 61: 282-286; Feb. 1961.

Chase, Clinton I. An Application of Levels of Concept Formation to Measurement of Vocabulary. *J. Ed. Res.* 55: 75-78; Oct. 1961.

Chase, Clinton I. The Control of Ability to Learn in the Comparison of Extreme Groups. *J. Ed. Res.* 57: 495-497; June 1964.

Chase, Sara E. Waste in Arithmetic. *Teach. Col. Rec.* 18: 360-370; Sept. 1917.

Chase, V. E. The Diagnosis and Treatment of Some Common Difficulties in Solving Arithmetic Problems. *J. Ed. Res.* 20: 335-342; Dec. 1929.

Chase, W. Linwood. Subject Preferences of Fifth-Grade Children. *El. Sch. J.* 50: 204-211; Dec. 1949.

Chase, W. Linwood and Wilson, Gilbert M. Preference Studies in Elementary School Social Studies. *J. Ed.* 140: 2-8; Apr. 1958.

Christensen, Donald J. The Effect of Discontinued Grade Reporting on Pupil Learning. *Arith. Teach.* 15: 724-726; Dec. 1968.

Christofferson, H. C. The Crucial Place of Seventh and Eighth Grade Mathematics in Education for Competency. *Sch. Sci. Math.* 41: 55-62; Jan. 1941.

Clark, Edward T. Sex Differences in the Perception of Academic Achievement Among Elementary School Children. *J. Psychol.* 67: 249-256; Nov. 1967.

Clark, Eileen. Number Experiences of Three-Year Olds. *Childhd. Ed.* 26: 247-250; Feb. 1950.

Clark, J. R. and Vincent, E. L. A Comparison of Two Methods of Arithmetic Problem Analysis. *Math. Teach.* 18: 226-233; Apr. 1925.

Clark, John R. and Vincent, E. Leona. A Study of the Effect of Checking Upon Accuracy in Addition. *Math. Teach.* 19: 65-71; Feb. 1926.

Clark, Leonard H. The Curriculum for Elementary Teachers in Sixty-Eight State Teachers Colleges. *J. Teach. Ed.* 6: 114-117; June 1955.

Cleland, D. L. and Toussaint, I. H. Interrelationships of Reading, Listening, Arithmetic Computation and Intelligence. *Read. Teach.* 15: 228-231; Jan. 1962.

Clem, Orlie M. and Hovey, Chester W. Comparative Achievement of Village-School Pupils and Rural-School Pupils. *El. Sch. J.* 34: 269-272; Dec. 1933.

Clemens, Paul B. and Neubauer, Paul F. A Supervision Project in Multiplication. *J. Ed. Res.* 18: 387-396; Dec. 1928.

Cleveland, Gerald Arthur and Bosworth, Dorothy L. A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement. *Arith. Teach.* 14: 383-387; May 1967.

Cluley, John B. A Study of the Relative Amounts of Transfer Resulting from Three Methods of Study. *J. Exp. Ed.* 1: 34-41; Sept. 1932.

Coers, Walter C. Comparative Achievement of White and Mexican Junior High School Pupils. *Peabody J. Ed.* 12: 157-162; Jan. 1935.

Coffing, Esther A. The Relationship Between Silent Reading Ability and Arithmetical Ability. *Sch. Sci. Math.* 41: 10-14; Jan. 1941.

Cohen, Walter; Hershkowitz, Aaron; and Chodack, Marjorie. Size Judgement at Different Distances as a Function of Age Level. *Child Develop.* 29: 473-479; Dec. 1958.

Collier, Myrtle. Learning to Multiply Fractions. *Sch. Sci. Math.* 22: 324-329; Apr. 1922.

Collings, Ellsworth. Learning the Fundamentals in the Activity Curriculum. *J. Exp. Ed.* 1: 309-315; June 1933.

Combs, Louise. Summary of Study of Certification Requirements in Mathematics for Elementary Teachers in the 50 States. *Am. Math Mon.* 70: 428-433; Apr. 1963.

Committee on the Undergraduate Program in Mathematics. Preparation in Mathematics for Elementary School Teachers. *Arith. Teach.* 14: 198-199; Mar. 1967.

Connor, William L. and Hawkins, Gertrude C. What Materials Are Most Useful to Children in Learning to Solve Problems? *Ed. Meth.* 16: 21-29; Oct. 1936.

Cook, Ruth Cathlyn. Vacation Retention of Fundamentals by Primary-Grade Pupils. *El. Sch. J.* 43: 214-219; Dec. 1942.

Cooke, Dennis H. Diagnostic and Remedial Treatment in Arithmetic. I. *Peabody J. Ed.* 9: 143-151; Nov. 1931. II. 10: 167-171; Nov. 1932.

Cooke, Dennis H. and Whitmore, B. E. Subject Combinations in Departmentalized Elementary Schools. *El. Sch. J.* 34: 526-532; Mar. 1934.

Corle, Clyde G. Thought Processes in Grade Six Problems. *Arith. Teach.* 5: 193-203; Oct. 1958.

Corle, Clyde G. A Study of the Quantitative Values of Fifth and Sixth Grade Pupils. *Arith. Teach.* 7: 333-340; Nov. 1960.

Corle, Clyde G. Estimates of Quantity by Elementary Teachers and College Juniors. *Arith. Teach.* 10: 347-352; Oct. 1963.

Counts, George Sylvester. Arithmetic Tests and Studies in the Psychology of Arithmetic. *Suppl. Ed. Monog.* 1: 1-125; 1917.

Court, S. R. A. Numbers, Time and Space in the First Five Years of the Child's Life. *Pedagog. Sem.* 27: 71-89; 1919.

Courtis, S. A. Measurement of Growth and Efficiency in Arithmetic. El. Sch. Teach. 10: 58-74, 177-199; 1909.

Courtis, S. A. Measurement of Growth and Efficiency in Arithmetic. El. Sch. Teach. 11: 171-185, 360-370, 528-539; 1911.

Coxford, Arthur F., Jr. Piaget: Number and Measurement. Arith. Teach. 10: 419-427; Nov. 1963.

Coxford, Arthur F., Jr. The Effects of Instruction on the Stage Placement of Children in Piaget's Seriation Experiments. Arith. Teach. 11: 4-9; Jan. 1964.

Cramer, John Francis. Australian Tests and American Pupils. El. Sch. J. 37: 17-24; Sept. 1936.

Creswell, John L. The Competence in Arithmetic of Prospective Georgia Elementary Teachers. Arith. Teach. 11: 248-250; Apr. 1964.

Creswell, John L. How Effective Are Modern Mathematics Workshops? Arith. Teach. 14: 205-208; Mar. 1967.

Crist, Robert L. Use of a Programed Multiplication Text Under Group-Paced and Individual-Paced Conditions. AV Comm. R. 14: 507-513; Winter 1966.

Cronin, Robert Emmet. The Effect of Varying Amounts of Traditional and Modern Mathematics Instruction Relative to Sex and Intellectual Ability on Both the Traditional and Modern Mathematics Achievement of Eighth Grade Pupils. Catholic Ed. R. 65: 548-549; Dec. 1967.

Cruickshank, William M. Arithmetic Vocabulary of Mentally Retarded Boys. J. Excep. Child. 13: 65-69, 91; Dec. 1946.

Cruickshank, William M. Arithmetic Work Habits of Mentally Retarded Boys. Am. J. Ment. Def. 52: 318-330; Apr. 1948.

Cruickshank, William M. Arithmetic Ability of Mentally Retarded Children. I. Ability to Differentiate Extraneous Materials From Needed Arithmetical Facts. J. Ed. Res. 42: 161-170; Nov. 1948.

Cruickshank, William M. Arithmetic Ability of Mentally Retarded Children. II. Understanding Arithmetic Processes. J. Ed. Res. 42: 279-288; Dec. 1948.

Curry, Robert L. Subject Preferences of Fifth-Grade Children. Peabody J. Ed. 41: 21-27; July 1963.

Cutts, N. E. and Moseley, Nicholas. Bright Children and the Curriculum. *Ed. Adm. and Sup.* 39: 168-173; Mar. 1953.

Dahle, Casper O. The Verbal Thought and Overt Behavior of Children During Their Learning of Long Division. *J. Exp. Ed.* 9: 1-8; Sept. 1940.

D'Augustine, Charles H. Topics in Geometry and Point Set Topology - A Pilot Study. *Arith. Teach.* 11: 407-412; Oct. 1964.

D'Augustine, Charles. Factors Related to Achievement With Selected Topics in Geometry and Topology. *Arith. Teach.* 13: 192-197; Mar. 1966.

Davis, O. L., Jr.; Carper, Barbara; and Crigler, Carolyn. The Growth of Pre-School Children's Familiarity With Measurement. *Arith. Teach.* 6: 186-190; Oct. 1959.

Davis, O. L. and Tracy, Neal H. Arithmetic Achievement and Instructional Grouping. *Arith. Teach.* 10: 12-17; Jan. 1963.

Davis, Robert A. and Rood, Edward J. Remembering and Forgetting Arithmetical Abilities. *J. Ed. Psychol.* 38: 216-222; Apr. 1947.

Dawson, Dan T. Number Grouping as a Function of Complexity. *El. Sch. J.* 54: 35-42; Sept. 1953.

Dawson, Dan T. and Ruddell, Arden K. An Experimental Approach to the Division Idea. *Arith. Teach.* 2: 6-9; Feb. 1955.

Dawson, Dan T. and Ruddell, Arden K. The Case for the Meaning Theory in Teaching Arithmetic. *El. Sch. J.* 55: 393-399; Mar. 1955.

Dean, Stuart E. Relation of Children's Subject Preferences to Their Achievement. *El. Sch. J.* 51: 89-92; Oct. 1950.

DeFrancis, John. Beginnings of Mathematical Education in Russia. *Arith. Teach.* 6: 6-11, 16; Feb. 1959.

DeFrancis, John. The Elementary Mathematics Program in Russia. *Sch. Sci. Math.* 60: 301-313; Apr. 1960.

Denmark, Thomas and Kalin, Robert. Suitability of Teaching Geometric Construction in Upper Elementary Grades - A Pilot Study. *Arith. Teach.* 11: 73-80; Feb. 1964.

DeVault, M. Vere; Houston, W. Robert; and Boyd, C. Claude. Do Consultant Services Make a Difference? *Sch. Sci. Math.* 63: 285-290; Apr. 1963.

Dewar, John A. Grouping for Arithmetic Instruction in Sixth Grade. *El. Sch. J.* 63: 266-269; Feb. 1963.

DeWeerdt, Esther Hurley. The Permanence of Improvement of Fifth Grade School Children in Certain Mental Functions. *J. Ed. Res.* 16: 127-131; Sept. 1927.

DeWeerdt, Esther Hurley. A Study of Improvability of Fifth-Grade School Children in Certain Mental Functions. *J. Ed. Psychol.* 18: 547-557; 1927.

Dexter, Clara E. Analysis of Written Problems in Recent Arithmetic Series. *Ed.* 65: 488-490; Apr. 1945.

Dickey, John W. The Value of Estimating Answers to Arithmetic Problems and Examples. *El. Sch. J.* 35: 24-31; Sept. 1934.

Dinitz, Simon; Kay, Barbara; and Reckless, Walter. Delinquency Proneness and School Achievement. *Ed. Res. B.* 36: 131-136; Apr. 10, 1957.

Distad, H. W. An Analysis of the Drill Provisions in Division of Decimals in Ten Arithmetic Series. *J. Ed. Res.* 27: 509-523; Mar. 1934.

Dobbs, Virginia and Neville, Donald. The Effect of Nonpromotion on the Achievement of Groups Matched From Retained First Graders and Promoted Second Graders. *J. Ed. Res.* 60: 472-475; July/Aug. 1967.

Doherty, Anne and Wunderlich, Richard A. Effect of Secondary Reinforcement Schedules on Performance of Problem-Solving Tasks. *J. Exp. Psychol.* 77: 105-108; May 1968.

Dominy, Mildred M. A Comparison: Textbooks, Domestic and Foreign. *Arith. Teach.* 10: 428-434; Nov. 1963.

Donaldson, P. R. Programmed Mathematics in Primary Schools. *Prim. Math.* 6: 31-37; June 1968.

Donovan, M. E. and Thorndike, Edward L. Improvement in a Practice Experiment Under School Conditions. *Am. J. Psychol.* 24: 426-428; July 1913.

Dooley, (Mother) M. Constance. The Relationship Between Arithmetic Research and the Content of Arithmetic Textbooks (1900-1957). *Arith. Teach.* 7: 178-183, 188; Apr. 1960.

Dooley, (Mother) M. Constance. The Relation Between Arithmetic Research and the Content of Arithmetic Textbooks, 1900-1957. *J. Exp. Ed.* 29: 315-318; Mar. 1961.

Dougherty, Florence. The Akron Experiment. Arith. Teach. 9: 446-448; Dec. 1962.

Douglass, Harl R. The Development of Number Concept in Children of Pre-School and Kindergarten Ages. J. Exp. Psychol. 8: 443-470; Dec. 1925.

Dresher, Richard. Training in Mathematics Vocabulary. Ed. Res. B. 13: 201-204; Nov. 14, 1934.

Drier, William H. Rural Background Found in Six Third-Grade Arithmetic Texts Published Between 1942 and 1946. J. Ed. Res. 42: 299-306; Dec. 1948.

Duncan, Roger. An Experimental Study of the Effect of Parents' Knowledge on Study Performance in S.M.S.G. Mathematics. J. Ed. Res. 58: 135-137; Nov. 1964.

Dunkley, M. E. Some Number Concepts of Disadvantaged Children. Arith. Teach. 12: 359-361; May 1965.

Durflinger, Glenn W. The Fundamentals Forgotten by College Students. J. Ed. Res. 49: 571-579; Apr. 1956.

Durr, William K. The Use of Arithmetic Workbooks in Relation to Mental Abilities and Selected Achievement Levels. J. Ed. Res. 51: 561-571; Apr. 1958.

Dutton, Wilbur H. Attitudes of Prospective Teachers Toward Arithmetic. El. Sch. J. 52: 84-90; Oct. 1951.

Dutton, Wilbur H. Measuring Attitudes Toward Arithmetic. El. Sch. J. 55: 24-31; Sept. 1954.

Dutton, Wilbur H. Attitudes of Junior High School Pupils Toward Arithmetic. Sch. R. 64: 18-22; Jan. 1956.

Dutton, Wilbur H. University Students' Comprehension of Arithmetical Concepts. Arith. Teach. 8: 60-64; Feb. 1961.

Dutton, Wilbur H. Attitude Change of Prospective Elementary School Teachers Toward Arithmetic. Arith. Teach. 9: 418-424; Dec. 1962.

Dutton, Wilbur H. Growth in Number Readiness in Kindergarten Children. Arith. Teach. 10: 251-255; May 1963.

Dutton, Wilbur H. Prospective Elementary School Teachers' Understanding of Arithmetical Concepts. J. Ed. Res. 58: 362-365; Apr. 1965.

Dutton, Wilbur H. Individualizing Instruction in Elementary School Mathematics for Prospective Teachers. *Arith. Teach.* 13: 227-231; Mar. 1966.

Dutton, Wilbur H. Teaching Time Concepts to Culturally Disadvantaged Primary-Age Children. *Arith. Teach.* 14: 358-364; May 1967.

Dutton, Wilbur H. Another Look at Attitudes of Junior High School Pupils Toward Arithmetic. *El. Sch. J.* 68: 265-268; Feb. 1968.

Dutton, Wilbur H. New Mathematics for Ethiopian Elementary Schools. *Arith. Teach.* 15: 115-125; Feb. 1968.

Dutton, Wilbur H. and Cheney, Augustine P. Pre-Service and In-Service Education of Elementary School Teachers in Arithmetic. *Arith. Teach.* 11: 192-198; Mar. 1964.

Dutton, Wilbur H. and Hammond, H. Reginald. Two In-Service Mathematics Programs for Elementary School Teachers. *Calif. J. Ed. R.* 17: 63-67; Mar. 1966.

Earhart, Eileen. Evaluating Certain Aspects of a New Approach to Mathematics in the Primary Grades. *Sch. Sci. Math.* 64: 715-720; Nov. 1964.

Easterday, Kenneth E. An Experiment With Low Achievers in Arithmetic. *Math. Teach.* 57: 462-468; Nov. 1964.

Eaton, Merrill T. The Value of the Dictaphone in Diagnosing Difficulties in Addition. *Ind. U. Sch. Ed. Bull.* 14: 5-10; Apr. 1938.

Eaton, Merrill T. A Survey of the Achievement in Arithmetic of 11,348 Sixth Grade Pupils in 486 Schools in Indiana. *Ind. U. Sch. Ed. Bull.* 20: 1-62; Mar. 1944.

Ebert, Reuben S. Generalization Abilities in Mathematics. *J. Ed. Res.* 39: 671-681; May 1946.

Echternacht, Charles and Gordon, Virginia. Breaking the Lock Step in Arithmetic. *Arith. Teach.* 9: 86-89; Feb. 1962.

Ediger, Marlow. The Student Teacher and Pupil Achievement in Elementary School Arithmetic. *Sch. Sci. Math.* 65: 697-700; Nov. 1965.

Eigen, Lewis D. A Comparison of Three Modes of Presenting a Programmed Instruction Sequence. *J. Ed. Res.* 55: 453-460; June/July 1962.

Elkind, David. The Development of Quantitative Thinking: A Systematic Replication of Piaget's Studies. *J. Genet. Psychol.* 98: 37-46; Jan. 1961.

Elkind, David. Children's Discovery of the Conservation of Mass, Weight, and Volume: Piaget Replication Study II. *J. Genet. Psychol.* 98: 219-227; Apr. 1961.

Elkind, David. The Development of the Additive Composition of Classes in the Child: Piaget Replication Study III. *J. Genet. Psychol.* 99: 51-57; July 1961.

Elkind, David. Discrimination, Seriation, and Numeration of Size and Dimensional Differences in Young Children: Piaget Replication Study VI. *J. Genet. Psychol.* 104: 275-296; Apr. 1964.

Ellsworth, Elmer E. Number Experiences of 390 Children From Grades 3-6 in an Urban Area. *Ed.* 61: 485-487; Apr. 1941.

Engelhart, Max D. The Relative Contribution of Certain Factors to Individual Differences in Arithmetical Problem Solving Ability. *J. Exp. Ed.* 1: 19-27; Sept. 1932.

Erhart, (Mother) M. Arithmetic for the Academically Talented. *Arith. Teach.* 7: 53-60; Feb. 1960.

Erickson, Leland H. Certain Ability Factors and Their Effect on Arithmetic Achievement. *Arith. Teach.* 5: 287-293; Dec. 1958.

Estes, Betsy Worth. Some Mathematical and Logical Concepts in Children. *J. Genet. Psychol.* 88: 219-222; Apr. 1956.

Estes, Betsy Worth. Judgment of Size in Relation to Geometric Shape. *Child Develop.* 32: 277-286; 1961.

Estes, Betsy and Combs, Ann. Perception of Quantity. *J. Genet. Psychol.* 108: 333-336; June 1966.

Evans, J. E. and Knoche, F. E. The Effects of Special Drill in Arithmetic as Measured by the Woody and the Courtis Arithmetic Tests. *J. Ed. Psychol.* 10: 263-276; May/June 1919.

Evans, John W., Jr. The Effect of Pupil Mobility Upon Academic Achievement. *Na. El. Prin.* 45: 18-22; Apr. 1966.

Evans, Roy. Remedial Cases in Arithmetic. Case 2. *Peabody J. Ed.* 7: 208-217; Jan. 1930.

Farley, Eugene S.; Frey, Albin J.; and Garland, Gertrude. Factors Related to the Grade Progress of Pupils. El. Sch. J. 34: 186-193; Nov. 1933.

Faulk, Charles J. How Well Do Pupils Estimate Answers. Arith. Teach. 9: 436-440; Dec. 1962.

Faulk, Charles J. and Landry, Thomas R. An Approach to Problem-Solving. Arith. Teach. 8: 157-160; Apr. 1961.

Fawcett, Will M. and others. Improving the Teaching of Mathematics: A Departmental Project. Calif. J. Ed. Res. 3: 159-163; Sept. 1952.

Fay, Leo C. The Relationship Between Specific Reading Skills and Selected Areas of Sixth Grade Achievement. J. Ed. Res. 43: 541-547; Mar. 1950.

Fedon, Peter J. The Role of Attitude in Learning Arithmetic. Arith. Teach. 5: 304-310; Dec. 1958.

Fehr, Howard F. The Mathematics Education of Youth. Teach. Col. Rec. 60: 179-190; Jan. 1959.

Fehr, Howard F.; McMeen, George; and Sobel, Max. Using Hand-Operated Computing Machines in Learning Arithmetic. Arith. Teach. 3: 145-150; Oct. 1956.

Feigenbaum, Kenneth D. Task Complexity and I.Q. as Variables in Piaget's Problem of Conservation. Child Develop. 34: 423-432; June 1963.

Feigenbaum, Kenneth D. and Sulkin, Howard. Piaget's Problem of Conservation of Discontinuous Quantities: A Teaching Experience. J. Genet. Psychol. 105: 91-97; July 1964.

Feinberg, Henry. Achievement of a Group of Socially Maladjusted Boys as Revealed by the Stanford Achievement Test. J. Soc. Psychol. 26: 203-212; 1947.

Feinberg, Henry. Achievement of a Group of Children in Foster Homes as Revealed by the Stanford Achievement Test. J. Genet. Psychol. 75: 293-303; 1949.

Feinberg, Henry. Achievement of Children in Orphan Homes as Revealed by the Stanford Achievement Test. J. Genet. Psychol. 85: 217-229; 1954.

Feldhake, Herbert J. Student Acceptance of the New Mathematics Programs. *Arith. Teach.* 13: 14-19; Jan. 1966.

Feldhusen, John; Check, John; and Klausmeier, Herbert J. Achievement in Subtraction. *El. Sch. J.* 61: 322-327; Mar. 1961.

Feldhusen, John F. and Klausmeier, Herbert J. Achievement in Counting and Addition. *El. Sch. J.* 59: 388-393; Apr. 1959.

Feldhusen, John and Klausmeier, Herbert J. Anxiety, Intelligence, and Achievement in Children of Low, Average and High Intelligence. *Child Develop.* 33: 403-409; 1962.

Feldhusen, John F.; Ramharter, Hazel; and Birt, Andrew T. The Teacher vs Programed Learning. *Wisc. J. Ed.* 95: 8-10; Nov. 1962.

Feldhusen, John F.; Thurston, John R.; and Benning, James J. Classroom Behavior, Intelligence, and Achievement. *J. Exp. Ed.* 36: 82-87; Winter 1967.

Fincher, Glen E. and Fillmer, H. T. Programmed Instruction in Elementary Arithmetic. *Arith. Teach.* 12: 19-23; Jan. 1965.

Finley, Carmen J. Arithmetic Achievement in Mentally Retarded Children: The Effects of Presenting the Problem in Different Contexts. *Am. J. Ment. Def.* 67: 281-286; Sept. 1962.

Finley, Carmen J. and Holmes, Jack A. Relative Importance of Curricular Areas for Grade Placement Deviations in Grade VI. *Calif. J. Ed. Res.* 7: 201-205, 216; Nov. 1956.

Finley, Carmen J. and Thompson, Jack M. A Comparison of the Achievement of Multi-Graded and Single-Graded Rural Elementary School Children. *J. Ed. Res.* 56: 471-475; May/June 1963.

Fisher, John J. Extent of Implementation of C.U.P.M. Level I Recommendations. *Arith. Teach.* 14: 194-197; Mar. 1967.

Fisher, S. Carolyn. Arithmetic and Reasoning in Children. *Pedagog. Sem.* 19: 48-77; Mar. 1912.

Fitzgerald, William M. On the Learning of Mathematics by Children. *Math. Teach.* 56: 517-521; Nov. 1963.

Flanders, Ned Allen. Verbalization and Learning in the Classroom. *El. Sch. J.* 48: 385-392; Mar. 1948.

Flournoy, Mary Frances. The Effectiveness of Instruction in Mental Arithmetic. *El. Sch. J.* 55: 148-153; Nov. 1954.

Flournoy, Frances. The Controversy Regarding the Teaching of Higher Decade Addition. Arith. Teach. 3: 170-173; Oct. 1956.

Flournoy, Frances. A Consideration of the Ways Children Think When Performing Higher-Decade Addition. El. Sch. J. 57: 204-208; Jan. 1957.

Flournoy, Frances. Developing Ability in Mental Arithmetic. Arith. Teach. 4: 147-150; Oct. 1957.

Flournoy, Frances. Children's Success With Two Methods of Estimating the Quotient Figure. Arith. Teach. 6: 100-104; Mar. 1959.

Flournoy, Frances. Providing Mental Arithmetic Experiences. Arith. Teach. 6: 133-138; Apr. 1959.

Flournoy, Frances. A Consideration of Pupils' Success With Two Methods for Placing the Decimal Point in the Quotient. Sch. Sci. Math. 59: 445-455; June 1959.

Flournoy, Frances. Meeting Individual Differences in Arithmetic. Arith. Teach. 7: 80-86; Feb. 1960.

Flournoy, Frances. Understanding Relationships: An Essential for Solving Equations. El. Sch. J. 64: 214-217; Jan. 1964.

Flournoy, Frances. Applying Basic Mathematical Ideas in Arithmetic. Arith. Teach. 11: 104-108; Feb. 1964.

Flournoy, Frances. A Study of Pupils' Understanding of Arithmetic in the Intermediate Grades. Sch. Sci. Math. 47: 325-333; Apr. 1967.

Flournoy, Frances. A Study of Pupils' Understanding of Arithmetic in the Primary Grades. Arith. Teach. 14: 481-485; Oct. 1967.

Flournoy, Frances. The Development of Arithmetic Understanding Tests for Primary and Intermediate Levels. J. Ed. Res. 62: 73-76; Oct. 1968.

Flournoy, Frances; Brandt, Dorothy; and McGregor, Johnnie. Pupil Understanding of the Numeration System. Arith. Teach. 10: 88-92; Feb. 1963.

Fogler, Sigmund. A Half-Pound of Cure Is Better Than None. Sch. Exec. 72: 62-63; June 1953.

Folsom, Mary. Teachers Look at Arithmetic Manuals. Arith. Teach. 7: 13-18; Jan. 1960.

Foran, T. G. and Lenaway, (Sister) M. Albert. Comparative Difficulty of Three Forms of Computation Tests. *J. Ed. Res.* 31: 568-571; Apr. 1938.

Foran, T. G. and Weber, (Sister) Magdalen Marie. An Experimental Study of the Relation of Homework to Achievement in Arithmetic. *Math. Teach.* 32: 212-214; May 1939.

Fowlkes, John G. A Report of a Controlled Study of the Learning of Multiplication by Third Grade Children. *J. Ed. Res.* 15: 181-189; Mar. 1927.

Freeman, Frank N. Grouped Objects as a Concrete Basis for the Number Idea. *El. Sch. Teach.* 12: 306-314; 1912.

Freyberg, P. S. Concept Development in Piagetian Terms in Relation to School Attainment. *J. Ed. Psychol.* 57: 164-168; June 1966.

Friebel, Allen C. Measurement Understandings in Modern School Mathematics. *Arith. Teach.* 14: 476-480; Oct. 1967.

Friedman, Kopple C. Time Concepts of Elementary-School Children. *El. Sch. J.* 44: 337-342; Feb. 1944.

Fulkerson, Elbert. How Well Do 158 Prospective Elementary Teachers Know Arithmetic? *Arith. Teach.* 7: 141-146; Mar. 1960.

Gabbert, M. L. Remedial Cases in Arithmetic, Case I. *Peabody J. Ed.* 147-155; Nov. 1929.

Gabel, Otto J. The Effect of Definite Versus Indefinite Quantitative Terms Upon the Comprehension and Retention of Social Studies Material. *J. Exp. Ed.* 9: 177-186; Dec. 1940.

Gagne, Robert and Bassler, Otto. Study of Retention of Some Topics of Elementary Nonmetric Geometry. *J. Ed. Psychol.* 54: 123-131; June 1963.

Gates, A. I.; Batchelder, M. I.; and Betzner, Jean. A Modern Systematic Versus an Opportunistic Method of Teaching. *Teach. Col. Rec.* 27: 679-700; Apr. 1926.

Gerberich, J. R. and Prall, C. E. Departmental Organization Versus Traditional Organization in the Intermediate Grades. *El. Sch. J.* 31: 671-677; May 1931.

Gibb, E. Glenadine. A Review of a Decade of Experimental Studies Which Compared Methods of Teaching Arithmetic. *J. Ed. Res.* 46: 603-608; Apr. 1953.

Gibb, E. Glenadine. Children's Thinking in the Process of Subtraction. *J. Exp. Ed.* 25: 71-80; Sept. 1956.

Gibb, E. Glenadine and Matala, Dorothy C. A Study of the Use of Special Teachers in Science and Mathematics in Grades 5 and 6. *Sch. Sci. Math.* 61: 569-572; Nov. 1961.

Gibb, E. Glenadine and Matala, Dorothy C. Study on the Use of Special Teachers of Science and Mathematics in Grades 5 and 6: Final Report. *Sch. Sci. Math.* 62: 565-585; Nov. 1962.

Gillet, Harry O. Placement of Arithmetic Topics. *J. NEA* 20: 199-200; June 1931.

Gilliland, A. R. and Humphreys, Dorothy Windes. Age, Sex, Method, and Interval as Variables in Time Estimation. *J. Genet. Psychol.* 63: 123-130; 1943.

Gilmary, Sister. Transfer Effects of Reading Remediation to Arithmetic Computation When Intelligence is Controlled and All Other School Factors Are Eliminated. *Arith. Teach.* 14: 17-20; Jan. 1967.

Glennon, Vincent J. A Study in Needed Redirection in the Preparation of Teachers of Arithmetic. *Math. Teach.* 42: 389-396; Dec. 1949.

Glennon, Vincent Joseph. A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels. *Harvard Ed. R.* 19: 62-64; Winter 1949.

Glick, Joseph and Wapner, Seymour. Development of Transitivity: Some Findings and Problems of Analysis. *Child Develop.* 39: 621-638; June 1968.

Goforth, Lillian. A Classroom Experiment in Teaching Reading and Arithmetic Through Games. *Ed. Meth.* 17: 231-235; Feb. 1938.

Goldberg, Susan. Probability Judgments by Preschool Children: Task Conditions and Performance. *Child Develop.* 37: 157-167; Mar. 1966.

Goldstein, Avram. Does Homework Help? A Review of Research. *El. Sch. J.* 60: 212-224; Jan. 1960.

Goldstone, Sanford; Boardman, William K.; and Lhaman, William T. Kinesthetic Cues in the Development of Time Concepts. *J. Genet. Psychol.* 93: 185-190; Oct. 1958.

Goodnow, Jacqueline J. and Bethon, Gloria. Piaget's Tasks: The Effects of Schooling and Intelligence. *Child Develop.* 37: 573-582; Sept. 1966.

Goodwin, William L. Effect of Selected Methodological Conditions on Dependent Measures Taken After Classroom Experimentation. *J. Ed. Psychol.* 57: 350-358; Dec. 1966.

Gorman, Frank H. The Arithmetic Vocabulary of the Elementary-School Teacher. *El. Sch. J.* 38: 373-379; Jan. 1938.

Gothberg, Laura C. The Mentally Defective Child's Understanding of Time. *Am. J. Ment. Def.* 53: 441-455; Jan. 1949.

Grafft, William D. and Ruddell, Arden K. Cognitive Outcomes of the S.M.S.G. Mathematics Program in Grades 4, 5, and 6. *Arith. Teach.* 15: 161-165; Feb. 1968.

Graham, William A. Individualized Teaching of Fifth and Sixth-Grade Arithmetic. *Arith. Teach.* 11: 233-234; Apr. 1964.

Grant, Albert. An Analysis of the Number Knowledge of First-Grade Pupils According to Levels of Intelligence. *J. Exp. Ed.* 7: 63-66; Sept. 1938.

Gratch, Gerald. The Development of the Expectation of the Nonindependence of Random Events in Children. *Child Develop.* 30: 217-229; 1959.

Graubard, Paul S. The Extent of Academic Retardation in a Residential Treatment Center. *J. Ed. Res.* 58: 78-80; Oct. 1964.

Gray, Howard A. and Hollingworth, Leta S. The Achievement of Gifted Children Enrolled and Not Enrolled in Special Opportunity Classes. *J. Ed. Res.* 24: 255-261; Nov. 1931.

Gray, Roland F. An Experiment in the Teaching of Introductory Multiplication. *Arith. Teach.* 12: 199-203; Mar. 1965.

Gray, Roland F. An Approach to Evaluating Arithmetic Understandings. *Arith. Teach.* 13: 187-191; Mar. 1966.

Greenblatt, E. L. An Analysis of School Subject Preferences of Elementary School Children of the Middle Grades. *J. Ed. Res.* 55: 554-560; Aug. 1962.

Greene, Harry A. Directed Drill in the Comprehension of Verbal Problems in Arithmetic. *J. Ed. Res.* 11: 33-40; Jan. 1925.

Greene, Harry A. A Critique of Remedial and Drill Materials in Arithmetic. *J. Ed. Res.* 21: 262-276; Apr. 1930.

Groff, Patrick. Arithmetic Achievement in Left-Handed Children. *Arith. Teach.* 9: 141-144; Mar. 1962.

Groff, Patrick J. Self-Estimates of Ability to Teach Arithmetic. Arith. Teach. 10: 479-480; Dec. 1963.

Grossnickle, Foster E. How to Estimate the Quotient Figure in Long Division. El. Sch. J. 32: 299-306; Dec. 1931.

Grossnickle, Foster E. Classification of the Estimations in Two Methods of Finding the Quotient in Long Division. El. Sch. J. 32: 595-604; Apr. 1932.

Grossnickle, Foster E. Practice Material in the Estimation of the Quotient in Long Division Found in Current Textbooks. El. Sch. J. 33: 130-141; Oct. 1932.

Grossnickle, Foster E. Cues in Division Problems Given in Nine Representative Textbooks in Arithmetic. El. Sch. J. 33: 452-461; Feb. 1933.

Grossnickle, Foster E. An Experiment With a One-Figure Divisor in Short and Long Division. I. El. Sch. J. 34: 496-506; Mar. 1934.

Grossnickle, Foster E. An Experiment with a One-Figure Divisor in Short and Long Division. II. El. Sch. J. 34: 590-599; Apr. 1934.

Grossnickle, Foster E. Methods of Estimating the Quotient in Long Division Used by Teacher-Training Students. El. Sch. J. 35: 448-453; Feb. 1935.

Grossnickle, Foster E. Reliability of Diagnosis of Certain Types of Errors in Long Division With a One-Figure Divisor. J. Exp. Ed. 4: 7-16; Sept. 1935.

Grossnickle, Foster E. To Check or Not to Check? El. Sch. J. 36: 35-39; Sept. 1935.

Grossnickle, F. E. Errors and Questionable Habits of Work in Long Division With a One-Figure Divisor. J. Ed. Res. 29: 355-368; Jan. 1936.

Grossnickle, F. E. The Incidence of Error in Division With a One-Figure Divisor When Short and Long Forms of Division Are Used. J. Ed. Res. 29: 509-511; Mar. 1936.

Grossnickle, Foster E. Transfer of Knowledge of Multiplication Facts to Their Use in Long Division. J. Ed. Res. 29: 677-685; May 1936.

Grossnickle, Foster E. Concepts in Social Arithmetic for the Eighth Grade Level. J. Ed. Res. 30: 475-488; Mar. 1937.

Grossnickle, Foster E. An Experiment With Two Methods of Estimation of the Quotient. El. Sch. J. 37: 668-677; May 1937.

Grossnickle, Foster E. The Effectiveness of Checking Subtraction by Addition. El. Sch. J. 38: 436-441; Feb. 1938.

Grossnickle, Foster E. Estimating the Quotient by Two Methods in Division With a Three-Figure Divisor. El. Sch. J. 39: 352-356; Jan. 1939.

Grossnickle, Foster E. Constancy of Error in Learning Division With a Two-Figure Divisor. J. Ed. Res. 33: 189-196; Nov. 1939.

Grossnickle, Foster E. Comparison of Achievement of Pupils Who Are Good and Poor in Learning Division With a Two-Figure Divisor. J. Ed. Res. 34: 346-351; Jan. 1941.

Grossnickle, Foster E. Types of Errors in Division of Decimals. El. Sch. J. 42: 184-194; Nov. 1941.

Grossnickle, Foster E. Kinds of Errors in Division of Decimals and Their Constancy. J. Ed. Res. 37: 110-117; Oct. 1943.

Grossnickle, Foster E. Some Factors Affecting a Test Score in Division of Decimals. J. Ed. Res. 37: 338-342; Jan. 1944.

Grossnickle, Foster E. Division Facts and Their Use in Estimation of Quotient With a Two-Figure Divisor. El. Sch. J. 45: 569-574; June 1945.

Grossnickle, Foster E. Illustrations in Arithmetic Textbooks. El. Sch. J. 47: 84-92; Oct. 1946.

Grossnickle, Foster E. Growth in Mathematical Ability Among Prospective Teachers of Arithmetic. Arith. Teach. 9: 278-279; May 1962.

Grossnickle, Foster E. and Snyder, John H. Constancy of Errors to Basic Facts in the Fundamental Operations in Arithmetic. J. Ed. Res. 32: 336-344; Jan. 1939.

Gruen, Gerald E. Experiences Affecting the Development of Number Conservation in Children. Child Develop. 36: 963-979; Dec. 1965.

Guertin, Wilson H. The Achievement and Abilities of a Group of Educable Mentally Handicapped. *J. Ed. Res.* 50: 145-150; Oct. 1956.

Guggenheim, Fred. Classroom Climate and the Learning of Mathematics. *Arith. Teach.* 8: 363-367; Nov. 1961.

Guiler, Walter S. Current Tendencies in Course of Study Making in Arithmetic. *Ed. Adm. and Sup.* 14: 46-51; 1928.

Guiler, Walter Scribner. Improving Computational Ability. *El. Sch. J.* 30: 111-116; Oct. 1929.

Guiler, Walter Scribner. Computational Errors Made By Teachers of Arithmetic. *El. Sch. J.* 33: 51-58; Sept. 1932.

Guiler, Walter Scribner. Improving Ability in Fractions. *Math. Teach.* 29: 232-240; May 1936.

Guiler, Walter Scribner. Difficulties in Fractions Encountered by Ninth-Grade Pupils. *El. Sch. J.* 46: 146-156; Nov. 1945.

Guiler, Walter Scribner. Difficulties in Decimals Encountered by Ninth-Grade Pupils. *El. Sch. J.* 46: 384-393; Mar. 1946.

Guiler, Walter Scribner. Difficulties in Percentage Encountered by Ninth-Grade Pupils. *El. Sch. J.* 46: 563-573; June 1946.

Guiler, W. S. and Edwards, Vernon. An Experimental Study of Methods of Instruction in Computational Arithmetic. *El. Sch. J.* 43: 353-360; Feb. 1943.

Gunderson, Agnes G. Nature and Amount of Arithmetic in Readers for Grades I and II. *El. Sch. J.* 36: 527-540; Mar. 1936.

Gunderson, Agnes G. Number Concepts Held by Seven-Year-Olds. *Math. Teach.* 33: 18-24; Jan. 1940.

Gunderson, Agnes G. Measurement in Arithmetic Textbooks for Grade Three. *Math. Teach.* 35: 117-121; Mar. 1942.

Gunderson, Agnes G. Thought Patterns of Young Children in Learning Multiplication and Division. *El. Sch. J.* 55: 453-461; Apr. 1955.

Gunderson, Agnes G. and Gunderson, Ethel. What Numbers Mean to Young Children. *Arith. Teach.* 6: 180-185; Oct. 1959.

Gunderson, Ethel and Gunderson, Agnes G. Fraction Concepts Held by Young Children. *Arith. Teach.* 4: 168-173; Oct. 1957.

Gunderson, Richard O. and Feldt, Leonard S. The Relationship of Differences Between Verbal and Nonverbal Intelligence Scores to Achievement. *J. Ed. Psychol.* 51: 115-121; Mar. 1960.

Hahn, H. H. and Thorndike, E. L. Some Results of Practice Addition Under School Conditions. *J. Ed. Psychol.* 5: 65-84; Feb. 1914.

Hall, Jack V. Oral Aids to Problem-Solving. *El. Sch. J.* 43: 220-224; Dec. 1942.

Hall, Jack V. Solving Verbal Arithmetic Problems Without Pencil and Paper. *El. Sch. J.* 48: 212-217; Dec. 1947.

Hall, Jack V. Mental Arithmetic: Misunderstood Terms and Meanings. *El. Sch. J.* 54: 349-353; Feb. 1954.

Halpern, Esther. The Effects of Incompatibility Between Perception and Logic in Piaget's Stage of Concrete Operations. *Child Develop.* 36: 491-497; June 1965.

Hambourger, James N. Grouping for Arithmetic Increases Pupil Growth. *Chicago Sch. J.* 45: 362-365; May 1964.

Hamilton, Dwight. An Experiment With an Individual Technique in Sixth Grade Arithmetic. *J. Ed. Res.* 18: 339-344; Nov. 1928.

Hamilton, E. W. Number Systems, Fad or Foundation? *Arith. Teach.* 8: 242-245; May 1961.

Hamza, Mukhtar. Retardation in Mathematics Amongst Grammar School Pupils. *Br. J. Ed. Psychol.* 22: 189-195; Nov. 1952.

Hanna, Paul R. Methods of Arithmetic Problem Solving. *Math. Teach.* 23: 442-450; Nov. 1930.

Hansen, Carl W. Factors Associated With Successful Achievement in Problem Solving in Sixth Grade Arithmetic. *J. Ed. Res.* 38: 111-118; Oct. 1944.

Hansen, Harvey C. Scholastic Achievement of Indian Pupils. *J. Genet. Psychol.* 50: 361-369; 1937.

Harap, Henry and Barrett, Ursala. Experimenting With Real Situations in Third Grade Arithmetic. *Ed. Meth.* 16: 188-192; Jan. 1937.

Harap, Henry and Mapes, Charlotte E. The Learning of Fundamentals in an Arithmetic Activity Program. *El. Sch. J.* 34: 515-525; Mar. 1934.

Harap, Henry and Mapes, Charlotte E. The Learning of Decimals in an Arithmetic Activity Program. *J. Ed. Res.* 29: 686-693; May 1936.

Hardgrove, Clarence E. and Jacobson, Bernard. C.U.P.M. Report on the Training of Teachers of Elementary School Mathematics. *Arith. Teach.* 11: 89-93; Feb. 1964.

Harding, Lowry W. and Bryant, Inez P. An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth Grade. *J. Ed. Res.* 37: 321-337; Jan. 1944.

Harlan, Charles L. Years in School and Achievements in Reading and Arithmetic. *J. Ed. Res.* 8: 145-149; Sept. 1923.

Harper, E. Harold. Elementary Teachers' Knowledge of Basic Arithmetic Concepts and Symbols. *Arith. Teach.* 11: 543-546; Dec. 1964.

Harrison, M. Lucile. The Nature and Development of Concepts of Time Among Young Children. *El. Sch. J.* 34: 507-514; Mar. 1934.

Harshman, Hardwick W.; Wells, David W.; and Payne, Joseph N. Manipulative Materials and Arithmetic Achievement in Grade 1. *Arith. Teach.* 9: 188-191; Apr. 1962.

Hart, Richard H. The Nongraded Primary School and Arithmetic. *Arith. Teach.* 9: 130-133; Mar. 1962.

Hartlein, Marion L. Use of Items With Coded Numbers for Measuring Understanding of Elementary Mathematical Concepts. *Arith. Teach.* 13: 540-545; Nov. 1966.

Harvey, Lois Fulcher. Improving Arithmetic Skills by Testing and Reteaching. *El. Sch. J.* 53: 402-409; Mar. 1953.

Harvey, Lois F. and Kyte, George C. Zero Difficulties in Multiplication. *Arith. Teach.* 12: 45-50; Jan. 1965.

Hebron, M. E. A Factorial Study of Learning a New Number System and Its Relation to Attainment, Intelligence and Temperament. *Br. J. Ed. Psychol.* 32: 38-45; Feb. 1962.

Heddons, James W. and Smith, Kenneth J. The Readability of Elementary Mathematics Books. *Arith. Teach.* 11: 466-468; Nov. 1964.

Heilman, J. D. Sex Differences in Intellectual Abilities. *J. Ed. Psychol.* 24: 47-62; Jan. 1933.

Henderson, George. Math Tests Analyzed. *Wisc. J. Ed.* 100: 16-17, 27; May 1968.

Henderson, Kenneth B. and Rollins, James H. A Comparison of Three Stratagems for Teaching Mathematical Concepts and Generalizations by Guided Discovery. Arith. Teach. 14: 503-508; Nov. 1967.

Henry, Lyle K. The Performance of Adults of Various Grade Levels in Specific Mental Functions: A Grade-by-Grade Comparison With School Pupils. J. Ed. Res. 33: 93-101; Oct. 1939.

Herman, Wayne L., Jr. How Intermediate Children Rank the Subjects. J. Ed. Res. 56: 435-436; Apr. 1963.

Hervey, Margaret A. Children's Responses to Two Types of Multiplication Problems. Arith. Teach. 13: 288-292; Apr. 1966.

Hicks, J. Allan and Stewart, Florence D. The Learning of Abstract Concepts of Size. Child Develop. 1: 195-203; Sept. 1930.

Hicks, Randall C. Elementary Series and Texts for Teachers - How Well Do They Agree? Arith. Teach. 15: 266-270; Mar. 1968.

Hicks, Randall C. and Perrodin, Alex F. Topics in Mathematics for Elementary School Teachers. Sch. Sci. Math. 47: 739-744; Nov. 1967.

Higgins, Conwell and Rusch, Reuben. Remedial Teaching of Multiplication and Division: Programmed Textbook Versus Workbook - A Pilot Study. Arith. Teach. 12: 32-38; Jan. 1965.

Hightower, H. W. Effect of Instructional Procedures on Achievement in Fundamental Operations in Arithmetic. Ed. Adm. and Sup. 40: 336-348; Oct. 1954.

Hightower, Howard W. Individual Differences. Ed. Adm. and Sup. 41: 458-461; Dec. 1955.

Hildreth, Gertrude. Results of Repeated Measurement of Pupil Achievement. J. Ed. Psychol. 21: 286-296; Apr. 1930.

Hildreth, Gertrude. The Success of Young Children in Number and Letter Construction. Child Develop. 3: 1-14; Mar. 1932.

Hildreth, Gertrude. Number Readiness and Progress in Arithmetic. J. Exp. Ed. 4: 1-6; Sept. 1935.

Hildreth, Gertrude. The Educational Achievement of Gifted Children. Child Develop. 9: 365-371; Dec. 1938.

Hinkelman, Emmet. Relationship of Intelligence to Elementary School Achievement. Ed. Adm. and Sup. 40: 176-179; Mar. 1955.

Hinkelman, Emmet A. A Study of the Principles Governing Fractions Known by the Fifth and Sixth Grade Children. Ed. Adm. and Sup. 42: 153-161; Mar. 1956.

Hirsch, Martin. Does Changing the Form of a Problem Affect Its Difficulty? High Points 33: 19-25; Dec. 1951.

Hobson, J. R. Physical Growth and School Achievement at Adolescence. Harvard Ed. R. 5: 154-164; June 1935.

Holinger, Dorothy. Helping the Non-Learner in Grade 1. Arith. Teach. 5: 15-24; Feb. 1958.

Hollingsworth, Mary R.; Lacey, Joy M.; and Shannon, J. R. School Subjects Which Elementary School Teachers Find Most Difficult and Those Which They Find Easiest to Teach. Ed. Meth. 10: 75-83; Nov. 1930.

Hollis, Loye Y. Why Teach Numeration? Arith. Teach. 11: 94-95; Feb. 1964.

Hollis, Loye Y. A Study to Compare the Effect of Teaching First and Second Grade Mathematics by the Cuisenaire-Gattegno Method With a Traditional Method. Sch. Sci. Math. 65: 683-687; Nov. 1965.

Holmes, Darrell. An Experiment in Learning Number Systems. Ed. Res. B. 28: 100-104, 111-112; Apr. 1949.

Holmes, Darrell and Harvey, Lois. An Evaluation of Two Methods of Grouping. Ed. Res. B. 35: 213-222; Nov. 14, 1956.

Holmes, Emma E. First Graders' Number Concept. Arith. Teach. 10: 195-196; Apr. 1963.

Holmes, Emma E. What Do Pre-First-Grade Children Know About Number? El. Sch. J. 63: 397-403; Apr. 1963.

Holmes, Jack A. and Finley, Carmen J. Relative Importance of Curricular Areas for Grade Placement Deviations in Grade V. Calif. J. Ed. Res. 6: 213-218; Nov. 1955.

Holmes, Jack A. and Finley, Carmen J. Under- and Over-Age Grade-Placements and School Achievement. J. Ed. Psychol. 48: 447-456; Nov. 1957.

Holowinsky, Ivan. The Relationship Between Intelligence (80-110 I.Q.) and Achievement in Basic Educational Skills. Training Sch. B. 58: 14-22; Feb. 1961.

Hoover, J. H. Motivated Drill Work in Third-Grade Arithmetic and Silent Reading. J. Ed. Res. 4: 200-211; Oct. 1921.

Hopkins, L. Thomas. Learning Essentials in an Activity Curriculum. J. Exp. Ed. 1: 298-303; June 1933.

Houston, W. Robert; Boyd, Claude C.; and DeVault, M. Vere. An In-Service Mathematics Education Program for Intermediate Grade Teachers. Arith. Teach. 8: 65-68; Feb. 1961.

Houston, W. Robert and DeVault, M. Vere. Mathematics In-Service Education: Teacher Growth Increases Pupil Growth. Arith. Teach. 10: 243-247; May 1963.

Howard, Charles F. Three Methods of Teaching Arithmetic. Calif. J. Ed. Res. 1: 25-29; Jan. 1950.

Howell, Etta; Dunn, Maude Wilson; and Stoker, Dora. Measuring the Skills in an Integrated Program. J. Exp. Ed. 1: 316-319; June 1933.

Hudgins, Bryce B. Effects of Group Experience on Individual Problem Solving. J. Ed. Psychol. 51: 37-42; Feb. 1960.

Hudgins, Bryce B. and Loftin, Lorraine. The Invisible Child in the Arithmetic Class: A Study of Teacher-Pupil Interaction. J. Genet. Psychol. 108: 143-152; Mar. 1966.

Hudgins, Bryce B. and Smith, Louis M. Group Structure and Productivity in Problem-Solving. J. Ed. Psychol. 57: 287-296; Oct. 1966.

Huettig, Alice and Newell, John M. Attitudes Toward Introduction of Modern Mathematics Program by Teachers With Large and Small Number of Years' Experience. Arith. Teach. 13: 125-130; Feb. 1966.

Hungerman, Ann D. Achievement and Attitude of Sixth-Grade Pupils in Conventional and Contemporary Mathematics Programs. Arith. Teach. 14: 30-39; Jan. 1967.

Hunnicutt, C. W. Arithmetic Attitudes. Math. Teach. 37: 33; Jan. 1944.

Hurlock, E. B. An Evaluation of Certain Incentives Used in School Work. J. Ed. Psychol. 16: 145-159; Mar. 1925.

Ilg, Frances and Ames, Louise B. Developmental Trends in Arithmetic. J. Genet. Psychol. 79: 3-28; Sept. 1951.

Inskeep, James E., Jr. Pre-Service Preparation to Teach Elementary School Mathematics. *Sch. Sci. Math.* 68: 43-51; Jan. 1968.

Inskeep, James and Rowland, Monroe. An Analysis of School Subject Preferences of Elementary School Children of the Middle Grades: Another Look. *J. Ed. Res.* 58: 225-228; Jan. 1965.

Irish, Elizabeth H. Improving Problem Solving by Improving Verbal Generalization. *Arith. Teach.* 11: 169-175; Mar. 1964.

Isaacs, Ann F. A Gifted Underachiever in Arithmetic - A Case Study. *Arith. Teach.* 6: 257-261; Nov. 1959.

Ivey, John O. Computation Skills: Results of Acceleration. *Arith. Teach.* 12: 39-42; Jan. 1965.

Jackson, Joseph. Analysis of a Team Teaching and of a Self-Contained Homeroom Experiment in Grades Five and Six. *J. Exp. Ed.* 32: 317-331; June 1964.

Jacobs, James N.; Beery, Althea; and Leinwohl, Judith. Evaluation of an Accelerated Arithmetic Program. *Arith. Teach.* 12: 113-119; Feb. 1965.

Jacobs, James N. and Bollenbacher, Joan. Teaching Seventh Grade Mathematics by Television. *Math. Teach.* 53: 543-547; Nov. 1960.

Jacobs, James N.; Bollenbacher, Joan; and Keiffer, Mildred. Teaching Seventh-Grade Mathematics by Television to Homogeneously Grouped Below-Average Pupils. *Math. Teach.* 54: 551-555; Nov. 1961.

Jaffe, Philip. Technique for Evaluating Arithmetic Courses of Study. *J. Exp. Ed.* 5: 351-355; June 1937.

Jamison, King W. An Experiment With a Variable Base Abacus. *Arith. Teach.* 11: 81-84; Feb. 1964.

Janes, Grace E. A Study of the Isolated Problems of Four Arithmetic Text Series. *Ed. Meth.* 16: 198-199; Jan. 1937.

Jarolimek, John and Foster, Clifford D. Quantitative Concepts in Fifth-Grade Social Studies Textbooks. *El. Sch. J.* 59: 437-442; May 1959.

Jarvis, Oscar T. Time Allotment Relationships to Pupil Achievement in Arithmetic. *Arith. Teach.* 10: 248-250; May 1963.

Jarvis, Oscar. An Analysis of Individual Differences in Arithmetic. *Arith. Teach.* 11: 471-473; Nov. 1964.

Jarvis, Oscar. Boy-Girl Ability Differences in Elementary School Arithmetic. *Sch. Sci. Math.* 64: 657-659; Nov. 1964.

Jarvis, Oscar T. Arithmetic and Science Time Allotment Practices in Intermediate Grades. *Sch. Sci. Math.* 66: 322-324; Apr. 1966.

Jerome, (Sister) Agnes. A Study of Twenty Slow Learners. *J. Ed. Res.* 53: 23-27; Sept. 1959.

Jersild, A. T. and others. An Evaluation of Aspects of the Activity Program in New York City Elementary Schools. *J. Exp. Ed.* 8: 166-207; Dec. 1939.

John, Lenore. The Effect of Using the Long-Division Form in Teaching Division by One-Digit Numbers. *El. Sch. J.* 30: 675-692; May 1930.

John, Lenore. Difficulties in Solving Problems in Arithmetic. *El. Sch. J.* 31: 202-215; Nov. 1930.

Johnson, Harry C. Problem-Solving in Arithmetic: A Review of the Literature. I. *El. Sch. J.* 44: 396-403; Mar. 1944. II. 44: 476-482; Apr. 1944.

Johnson, Harry C. The Effect of Instruction in Mathematical Vocabulary Upon Problem Solving in Arithmetic. *J. Ed. Res.* 38: 97-110; Oct. 1944.

Johnson, J. T. The Efficiency of Decomposition Compared With That of Equal Additions as a Technique in Subtraction of Whole Numbers. *Math. Teach.* 24: 5-13; Jan. 1931.

Johnson, John T. Grade Placement of Mathematics Units. *Chicago Sch. J.* 22: 171-175; Apr. 1941.

Johnson, J. T. An Evaluation of Research on Gradation in the Field of Arithmetic. *J. Ed. Res.* 37: 161-173; Nov. 1943.

Johnson, John T. Can Concepts in Elementary Mathematics Be Developed? *Sch. Sci. Math.* 44: 146-154; Feb. 1944.

Johnson, J. T. On the Nature of Problem-Solving in Arithmetic. *J. Ed. Res.* 43: 110-115; Oct. 1949.

Johnson, J. T. The Use of a Ruler in Teaching Place Value in Numbers. *Math. Teach.* 45: 264-266; Feb. 1952.

Johnson, J. T. Decimal Versus Common Fractions. *Arith. Teach.* 3: 201-203; Nov. 1956.

Johnson, Mauritz and Scriven, Eldon. Class Size and Achievement Gains in Seventh- and Eighth-Grade English and Mathematics. *Sch. R.* 75: 300-310; Aug. 1967.

Johnson, Richard T. and Anderson, Robert A. Arithmetic Achievement in England and Central Minnesota. *Arith. Teach.* 11: 176-180; Mar. 1964.

Johnston, A. Montgomery. A Survey of Teaching Practices - Arithmetic in Tennessee in the First Through Eighth Grades. *Arith. Teach.* 9: 425-432; Dec. 1962.

Jones, Daisy Marvel. An Experiment in Adaptation to Individual Differences. *J. Ed. Psychol.* 39: 237-272; May 1948.

Jones, Emily. Historical Conflict - Decimal Versus Vulgar Fractions. *Arith. Teach.* 7: 184-188; Apr. 1960.

Josephina, (Sister). Differences in Arithmetic Performance. *Arith. Teach.* 6: 152-153, 166; Apr. 1959.

Josephina, (Sister). Mental Arithmetic in Today's Classroom. *Arith. Teach.* 7: 199-200; Apr. 1960.

Josephina, (Sister). A Study of Spatial Abilities of Preschool Children. *Arith. Teach.* 11: 557-560; Dec. 1964.

Josephina, (Sister). Quantitative Thinking of Preschool Children. *Arith. Teach.* 12: 54-55; Jan. 1965.

Joyce, Bruce and Elizabeth. Studying Issues in Mathematics Instruction. *Arith. Teach.* 11: 303-307; May 1964.

Judd, Charles Hubbard. Psychological Analysis of the Fundamentals of Arithmetic. *Suppl. Ed. Monog.* 32: 1-121; Feb. 1927.

Kalin, Robert. The Use of Programed Instruction in Teaching an Advanced Mathematical Topic. *Arith. Teach.* 9: 160-161; Mar. 1962.

Kane, Robert B. Attitudes of Prospective Elementary School Teachers Toward Mathematics and Three Other Subject Areas. *Arith. Teach.* 15: 169-175; Feb. 1968.

Kapos, Ervin and others. Schoolroom Motivation: I. Two Studies of Quantity and Pattern of Verbal Reinforcement as Related to Performance on a Routine Task. *Ind. U. Sch. Ed. Bull.* 33: 1-43; Jan. 1957.

Kapos, Ervin and others. Schoolroom Motivation: II. Two Studies of Quantity and Pattern of Verbal Reinforcement as Related to a Measure of Drive on a Routine Task. Ind. U. Sch. Ed. Bull. 33: 1-40; Mar. 1957.

Kaprelian, George. Attitudes Toward a Television Program-Patterns in Arithmetic. Arith. Teach. 8: 408-412; Dec. 1961.

Karstens, Harry. Estimating the Quotient Figure. J. Ed. Res. 38: 522-528; Mar. 1945.

Karstens, Harry. Checking the Estimate in Long Division. J. Ed. Res. 40: 52-56; Sept. 1946.

Kathleen, (Sister) M. Some Results of Remedial Instruction Following the Use of Diagnostic Arithmetic Tests. Catholic Ed. R. 24: 19-27; Jan. 1926.

Kazienka, Louis W. Beginner Grade Influence on School Progress. Ed. Adm. and Sup. 40: 219-228; Apr. 1954.

Keislar, Evan R. The Development of Understanding in Arithmetic by a Teaching Machine. J. Ed. Psychol. 50: 247-253; Dec. 1959.

Kelly, F. J. The Results of Three Types of Drill on the Fundamentals of Arithmetic. J. Ed. Res. 2: 693-700; Nov. 1920.

Kennedy, Leonard M. and Alves, Robert. In-Service Education for Elementary School Mathematics Teachers: Responses to Nine Questions. Arith. Teach. 11: 506-509; Nov. 1964.

Kenney, Russell A. Mathematical Understandings of Elementary School Teachers. Arith. Teach. 12: 431-442; Oct. 1965.

Kenney, Russell A. and Stockton, Jesse D. An Experimental Study in Teaching Percentage. Arith. Teach. 5: 294-303; Dec. 1958.

Keough, John J. The Relationship of Socio-Economic Factors and Achievement in Arithmetic. Arith. Teach. 7: 231-237; May 1960.

Keys, Noel and Lawson, J. V. Summer Versus Winter Gains in School Achievement. Sch. and Soc. 46: 541-544; Oct. 23, 1937.

Kingston, Albert J., Jr. The Relationship of First-Grade Readiness to Third- and Fourth-Grade Achievement. J. Ed. Res. 56: 61-67; Oct. 1962.

Kipps, Carol. Elementary Teachers' Ability to Understand Concepts Used in New Mathematics Curricula. Arith. Teach. 15: 367-370; Apr. 1968.

Kirby, - - -. Practice in the Case of School Children. Columbia Studies in Ed. 3: 1-98; 1913.

Kircher, H. W. A Study of Percentage in Grade VIII-A. El. Sch. J. 27: 281-289; Dec. 1926.

Kirkpatrick, E. A. An Experiment in Memorizing Versus Incidental Learning. J. Ed. Psychol. 5: 405-412; Sept. 1914.

Klausmeier, Herbert J. Effects of Accelerating Bright Older Elementary Pupils: A Follow Up. J. Ed. Psychol. 54: 165-171; 1963.

Klausmeier, Herbert J. Gear Learning Activities to Achievement Level of Students. Wisc. J. Ed. 96: 28-29; Feb. 1964.

Klausmeier, Herbert. Improving Problem Solving. Wisc. J. Ed. 96: 15-16; Mar. 1964.

Klausmeier, Herbert J. Accelerating Bright Older Children. Wisc. J. Ed. 96: 19-20, 27; May 1964.

Klausmeier, Herbert J.; Beeman, Alan; and Lehmann, Irvin J. Comparison of Organismic Age and Regression Equations in Predicting Achievements in Elementary School. J. Ed. Psychol. 49: 182-186; 1958.

Klausmeier, Herbert J. and Check, John. Retention and Transfer in Children of Low, Average, and High Intelligence. J. Ed. Res. 55: 319-322; Apr. 1962.

Klausmeier, Herbert J. and Feldhusen, John F. Retention in Arithmetic Among Children of Low, Average, and High Intelligence at 117 Months of Age. J. Ed. Psychol. 50: 88-92; Apr. 1959.

Klausmeier, Herbert J. and Laughlin, Leo J. Behaviors During Problem Solving Among Children of Low, Average, and High Intelligence. J. Ed. Psychol. 52: 148-152; June 1961.

Klausmeier, Herbert J. and Ripple, Richard E. Effects of Accelerating Bright Older Pupils From Second to Fourth Grade. J. Ed. Psychol. 53: 93-100; 1962.

Klugman, Samuel F. Cooperative Versus Individual Efficiency in Problem-Solving. J. Ed. Psychol. 35: 91-100; Feb. 1944.

Knight, Elton E. A Study of Double Grades in New Haven City Schools. J. Exp. Ed. 7: 11-18; Sept. 1938.

Knight, F. B. The Superiority of Distributed Practice in Drill in Arithmetic. J. Ed. Res. 15: 157-163; Mar. 1927.

Knight, F. B. A Report of Four Studies in Arithmetic. *J. Ed. Res.* 30: 325-339; Jan. 1937. Study 1: (McNamara, J. J.) An Experimental Study of the Relative Perceptual Value of Various Configurations of the Numbers Six to Twelve, Inclusive.

Knight, F. B. A Report of Four Studies in Arithmetic. *J. Ed. Res.* 30: 325-339; Jan. 1937. Study 2: (Harper, Jeanette) Tri-Dimensional Space as an Aid in Teaching Primary Arithmetic.

Knight, F. B. A Report of Four Studies in Arithmetic. *J. Ed. Res.* 30: 325-339; Jan. 1937. Study 3: (Scott, Mildred) The Sociology of Verbal Problems in Fourth Grade Arithmetic.

Knight, F. B. A Report of Four Studies in Arithmetic. *J. Ed. Res.* 30: 325-329; Jan. 1937. Study 4: (Ernst, W. M.) Appraisal of Illustrations in Fourth-Grade Arithmetic Texts.

Knight, F. B. and Ford, E. Temporary Lapses in Ability and Error in Arithmetic. *El. Sch. J.* 32: 111-124; Oct. 1931.

Knight, F. B. and Setzafandt, A. O. H. Transfer Within a Narrow Mental Function. *El. Sch. J.* 24: 780-787; June 1924.

Knipp, Minnie B. An Investigation of Experimental Studies Which Compare Methods of Teaching Arithmetic. *J. Exp. Ed.* 13: 23-30; Sept. 1944.

Koch, Elmer A., Jr. Homework in Arithmetic. *Arith. Teach.* 12: 9-13; Jan. 1965.

Koenker, Robert H. Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division. *J. Ed. Res.* 35: 578-586; Apr. 1942.

Koenker, Robert H. Arithmetic Readiness at the Kindergarten Level. *J. Ed. Res.* 42: 218-223; Nov. 1948.

Kohler, Richard C. Use of Arts Activities With Reading, Science, and Arithmetic. *El. Sch. J.* 52: 355-359; Feb. 1952.

Kolb, John R. Effects of Relating Mathematics to Science Instruction on the Acquisition of Quantitative Science Behaviors. *J. Res. Sci. Tchg.* 5: 174-182; June 1967.

Kolson, Clifford J. The Oral Arithmetic Vocabulary of Kindergarten Children. *Arith. Teach.* 10: 81-83; Feb. 1963.

Koontz, William F. A Study of Achievement as a Function of Homogeneous Grouping. *J. Exp. Ed.* 30: 249-253; Dec. 1961.

Kramer, Klaas. Arithmetic Achievement in Iowa and The Netherlands. *El. Sch. J.* 59: 258-263; Feb. 1959.

Kreismer, Clifford R. Glen Rock Scores on a Canadian Test. *Arith. Teach.* 5: 216-217; Oct. 1958.

Krich, Percy. Grade Placement and Meaningful Learning. *Sch. Sci. Math.* 64: 131-137; Feb. 1964.

Krich, Percy. Meaningful vs. Mechanical Method. Teaching Division of Fractions by Fractions. *Sch. Sci. Math.* 64: 697-708; Nov. 1964.

Kulp, C. L. A Study of the Relative Effectiveness of Two Types of Standard Arithmetic Practice Materials. *J. Ed. Res.* 22: 381-387; Dec. 1930.

Kyte, George C. and Fornwalt, James E. A Comparison of Superior Children With Normal Children in the Rate Mastery of the Multiplication of Fractions. *J. Ed. Res.* 60: 346-350; Apr. 1967.

Lambert, William W.; Solomon, Richard L.; and Watson, Peter D. Reinforcement and Extinction as Factors in Size Estimation. *J. Exp. Psychol.* 39: 637-641; 1949.

Lane, Howard A. and Witty, Paul A. The Educational Attainment of Delinquent Boys. *J. Ed. Psychol.* 25: 695-702; 1934.

Layton, W. I. Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers. *Math. Teach.* 44: 551-556; Dec. 1951.

LeBaron, Walter A. A Study of Teachers' Opinions in Methods of Teaching Arithmetic in the Elementary School. *J. Ed. Res.* 43: 1-9; Sept. 1949.

Lee, Doris M. A Study of Specific Ability and Attainment in Mathematics. *Br. J. Ed. Psychol.* 25: 178-189; Nov. 1955.

Lehew, Charmon. The Performance of Four- and Five-Year-Old Children in Operation Head Start on Selected Arithmetic Abilities. *Arith. Teach.* 15: 53-59; Jan. 1968.

Leibowitz, Sarah L. Fryer. The Motivational Effect of Value Symbols and Competition Upon Problem-Solving Behavior in Children. *J. Genet. Psychol.* 108: 327-332; June 1966.

Lerch, Harold H. Arithmetic Instruction Changes Pupils' Attitudes Toward Arithmetic. *Arith. Teach.* 7-8: 117-119; Mar. 1961.

Lerch, Harold H. Fourth-Grade Pupils Study a Number System With Base Five. *J. Ed. Res.* 57: 59-63; Oct. 1963.

Lerch, Harold H. and Hamilton, Helen. A Comparison of a Structured-Equation Approach to Problem Solving With a Traditional Approach. *Sch. Sci. Math.* 66: 241-246; Mar. 1966.

Lerch, Harold H. and Kelly, Francis J. A Mathematics Program for Slow Learners at the Junior High Level. *Arith. Teach.* 13: 232-236; Mar. 1966.

Lerch, Harold H. and Mangrum, Charles T., II. Instructional Aids Suggested by Textbook Series. *Arith. Teach.* 12: 543-546; Nov. 1965.

Lessenger, W. E. Reading Difficulties in Arithmetical Computation. *J. Ed. Res.* 11: 287-291; Apr. 1925.

Lewis, Eunice and Plath, Ernest C. Plus Work for "Plus" Pupils. *Arith. Teach.* 6: 251-256; Nov. 1959.

Lewis, Michael. Probability Learning in Young Children: The Binary Choice Paradigm. *J. Genet. Psychol.* 108: 43-48; Mar. 1966.

Lindgren, Henry Clay; Silva, Ina; Faraco, Italia; and Da Rocha, Nadir S. Attitudes Toward Problem Solving as a Function of Success in Arithmetic in Brazilian Elementary Schools. *J. Ed. Res.* 58: 44-45; Sept. 1964.

Long, Louis. Size Discrimination in Children. *Child Develop.* 12: 247-254; Sept. 1941.

Long, Louis and Welch, Livingston. The Development of the Ability to Discriminate and Match Numbers. *J. Genet. Psychol.* 59: 377-387; 1941.

Lucow, William H. The Cuisenaire Method. *Arith. Teach.* 10: 435-438; Nov. 1963.

Lucow, William H. An Experiment With the Cuisenaire Method in Grade Three. *Am. Ed. Res.* 1: 159-167; May 1964.

Lutes, O. S. Where Pupils Fail in Verbal Problems. *J. Ed. Res.* 13: 71-72; Jan. 1926.

Lutes, Olin S. An Evaluation of Three Techniques for Improving Ability to Solve Arithmetic Problems. *U. Iowa Monog. in Ed.* 6: 1-41; June 1926.

Lyda, W. J. Direct, Practical Experiences in Mathematics and Success in Solving Realistic Verbal "Reasoning" Problems in Arithmetic. Math. Teach. 40: 166-167; Apr. 1947.

Lyda, Wesley J. and Church, Ruby Summers. Direct, Practical Arithmetical Experiences and Success in Solving Realistic Verbal "Reasoning" Problems in Arithmetic. J. Ed. Res. 57: 530-533; July/Aug. 1964.

Lyda, W. J. and Duncan, Frances M. Quantitative Vocabulary and Problem Solving. Arith. Teach. 14: 289-291; Apr. 1967.

Lyda, Wesley J. and Morse, Evelyn Clayton. Attitudes, Teaching Methods, and Arithmetic Achievement. Arith. Teach. 10: 136-138; Mar. 1963.

Lyda, W. J. and Robinson, Verna A. Quantitative Concepts in Selected Social Studies Textbooks for Second Grade. El. Sch. J. 65: 159-162; Dec. 1964.

Lyda, W. J. and Taylor, Margaret D. Facilitating an Understanding of the Decimal Numeration System Through Modular Arithmetic. Arith. Teach. 11: 101-103; Feb. 1964.

Lynn, R. Temperamental Characteristics Related to Disparity of Attainment in Reading and Arithmetic. Br. J. Ed. Psychol. 27: 62-67; Feb. 1957.

MacLatchy, Josephine. Addition in the First Grade. Ed. Res. B. 9: 391-393, 406; Oct. 8, 1930.

MacLatchy, Josephine H. Number Ideas of Young Children. Childhd. Ed. 7: 59-66; Oct. 1930.

MacLatchy, Josephine H. A Phase of First-Grade Readiness. Ed. Res. B. 10: 377-380; Oct. 14, 1931.

MacLatchy, Josephine H. Counting and Addition. Ed. Res. B. 9: 96-100; Feb. 17, 1932.

MacLatchy, Josephine H. Another Measure of the Difficulty of Addition Combinations. Ed. Res. B. 12: 57-61; Mar. 8, 1933.

MacLatchy, Josephine H. The Pre-School Child's Familiarity With Measurement. Ed. 71: 479-482; Apr. 1951.

MacLatchy, Josephine H. and Hummel, Eugenie. Arithmetic With Understanding. Ed. Res. B. 21: 227-238, 246; Nov. 10, 1942.

McAulay, J. D. What Understanding Do Second Grade Children Have of Time Relationships? *J. Ed. Res.* 54: 312-314; Apr. 1961.

McCandless, Boyd R. and Castaneda, Alfred. Anxiety in Children, School Achievement, and Intelligence. *Child Develop.* 27: 379-382; Sept. 1956.

McCann, Bernice and Dalrymple, Charles O. Teaching the New Arithmetic in a Third Grade. *Ed.* 65: 493-496; Apr. 1945.

McDowell, Louise K. Number Concepts and Preschool Children. *Arith. Teach.* 9: 433-435; Dec. 1962.

McGrath, Robert T. Achievement in One-Room Schools. *Sch. Exec.* 56: 438-439; July 1937.

McKibben, Margaret J. Elementary School Science and Mathematics Education in Western Europe. *Sch. Sci. Math.* 61: 404-417; June 1961.

McIntosh, H. W. and Schrammel, H. E. A Comparison of the Achievement of Eighth Grade Pupils in Rural Schools and in Graded Schools. *El. Sch. J.* 31: 301-306; Dec. 1930.

McLaughlin, Jack W. A New Approach to an Old Problem. *Arith. Teach.* 8: 112-116; Mar. 1961.

McLaughlin, Katherine L. Number Ability of Preschool Children. *Childhd. Ed.* 11: 348-353; May 1935.

McMahon, Ottis. A Study of the Ability of Fifth Grade Children to Read Various Types of Material. *Peabody J. Ed.* 20: 228-233; Jan. 1943.

McMaster, Dale. Case Studies of Failing Pupils in Seventh Year Reading and Arithmetic. *Pittsburgh U. Sch. Ed. J.* 5: 39-46; Dec. 1929.

McNeil, John D. Prompting Versus Intermittent Confirmation in the Learning of a Mathematical Task. *Arith. Teach.* 12: 533-536; Nov. 1965.

Maertens, Norbert. Effects of Arithmetic Homework Upon the Attitudes of Third Grade Pupils Toward Certain School-Related Structures. *Sch. Sci. Math.* 68: 657-662; Oct. 1968.

Malone, William H. and Freel, Eugene L. A Preliminary Study of the Group Attitudes of Junior and Senior High School Students Toward Mathematics. *J. Ed. Res.* 47: 599-608; Apr. 1954.

Manuel, H. T. A Comparison of Spanish-Speaking and English-Speaking Children in Reading and Arithmetic. *J. Appl. Psychol.* 19: 189-202; Apr. 1935.

Marshall, Helen R. Transposition in Children as a Function of Age and Knowledge. *J. Genet. Psychol.* 108: 65-69; Mar. 1966.

Mary Jacqueline, (Sister). An Experiment in Remedial Teaching in Arithmetic. *El. Sch. J.* 41: 748-755; June 1941.

Mascho, George. Familiarity With Measurement. *Arith. Teach.* 8: 164-167; Apr. 1961.

Mays, W. Logic for Juniors. *Teach. Arith.: Br. Elem. Math. J.* 3: 3-10; Autumn 1965.

Mead, Cyrus D. and Sears, Isabel. Additive Subtraction and Multiplicative Division Tested. *J. Ed. Psychol.* 7: 261-270; May 1916.

Mead, C. D. and Johnson, C. W. Teaching Practice Material in the Fundamentals. *J. Ed. Psychol.* 9: 287-297; May 1918.

Meadowcroft, Bruce A. Comparison of Two Methods of Using Programmed Learning. *Arith. Teach.* 12: 422-425; Oct. 1965.

Meadowcroft, Bruce A. The Effects on Conventionally Taught Eighth-Grade Math Following Seventh-Grade Programmed Math. *Arith. Teach.* 12: 614-616; Dec. 1965.

Meconi, L. J. Concept Learning and Retention in Mathematics. *J. Exp. Ed.* 36: 51-57; Fall 1967. (see: Meconi, L. J. The Mathematically Gifted Student and Discovery Learning. *Math. Teach.* 60: 862-865; Dec. 1967.)

Meddleton, Ivor G. An Experimental Investigation Into the Systematic Teaching of Number Combinations in Arithmetic. *Br. J. Ed. Psychol.* 26: 117-127; June 1956.

Melson, Ruth. How Well are Colleges Preparing Teachers for Modern Mathematics? *Arith. Teach.* 12: 51-53; Jan. 1965.

Merton, Elda L. and Banting, G. O. Remedial Work in Arithmetic. *Yrbk. of Dept. El. Sch. Prin.* 2: 395-429; 1923.

Messick, A. I. Effect of Certain Types of Speed Drills in Arithmetic. *Math. Teach.* 19: 104-109; Feb. 1926.

Messick, Samuel J. and Solley, Charles M. Probability Learning in Children: Some Exploratory Studies. *J. Genet. Psychol.* 90: 23-32; Jan. 1957.

Messler, Dorothy L. A Study of Pupil Age and Achievement in Eighth-Grade Algebra. Math. Teach. 54: 561-564; Nov. 1961.

Metfessel, Newton S. The Sangus Experiment in Multi-Grade Grouping. Calif. J. Ed. Res. 11: 155-158; Sept. 1960.

Metter, Harry L. Trends in the Emphasis on Various Topics of Arithmetic Since 1860. El. Sch. J. 34: 767-775; June 1934.

Miller, G. H. How Effective is the Meaning Method? Arith. Teach. 4: 45-49; Mar. 1957.

Miller, G. H. How Much Time for Arithmetic? Arith. Teach. 5: 256-259; Nov. 1958.

Miller, G. H. Time Spent on Arithmetic in Foreign Countries and in the United States. Arith. Teach. 7: 217-221; May 1960.

Miller, G. H. Shall We Change Our Arithmetic Program? Arith. Teach. 9: 193-197; Apr. 1962.

Miller, G. H. Geometry in the Elementary Grades: A Comparative Study of Greek Mathematics Education. Arith. Teach. 11: 85-88; Feb. 1964.

Miller, Jack W. An Experimental Comparison of Two Approaches to Teaching Multiplication of Fractions. J. Ed. Res. 57: 468-471; May/June 1964.

Millman, Jason and Johnson, Mauritz, Jr. Relation of Section Variance to Achievement Gains in English and Mathematics in Grades 7 and 8. Am. Ed. Res. J. 1: 47-51; Jan. 1964.

Mitchell, Claude. The Specific Type of Problem in Arithmetic Versus the General Type of Problem. El. Sch. J. 29: 594-596; Apr. 1929.

Mitchell, Claude. Problem Analysis and Problem-Solving in Arithmetic. El. Sch. J. 32: 464-466; Feb. 1932.

Moench, Laurel. Individualized Practice in Arithmetic - A Pilot Study. Arith. Teach. 9: 321-329; Oct. 1962.

Monroe, Walter S. The Ability to Place the Decimal Point in Division. El. Sch. J. 18: 287-293; Dec. 1917.

Monroe, W. S. The Derivation of Reasoning Tests in Arithmetic. Sch. and Soc. 8: 295-299, 324-329; Sept. 7, 14, 1918.

Monroe, Walter S. How Pupils Solve Problems in Arithmetic. U. Ill. Bull. 44: 1-30, 1929.

Monroe, Walter S. and Clark, John A. The Teacher's Responsibility for Devising Learning Exercises in Arithmetic. U. Ill. Bull. 31: 1-92; 1926.

Monroe, Walter S. and Engelhart, Max D. The Effectiveness of Systematic Instruction in Reading Verbal Problems in Arithmetic. El. Sch. J. 33: 377-381; Jan. 1933.

Montague, David O. Arithmetic Concepts of Kindergarten Children in Contrasting Socio-Economic Areas. El. Sch. J. 64: 393-397; Apr. 1964.

Morfitt, Margaret D. K. Comparison of Individual-Concrete Methods and Class Methods in the Teaching of Arithmetic. Br. J. Ed. Psychol. 7: 196-203; June 1937.

Morgan, L. D. Specific vs. General Drill in the Fundamentals of Arithmetic. Sch. Sci. Math. 29: 528-529; May 1929.

Morrison, Maria Paul. Mass Method Versus Individual Method in Teaching Multiplication to Fourth Grade Pupils. Ed. 57: 345-347; Feb. 1937.

Morton, Dan M. Number Forms and Arithmetical Ability in Children. Br. J. Ed. Psychol. 6: 58-73; Feb. 1936.

Morton, John A. A Study of Children's Mathematical Interest Questions as a Clue to Grade Placement of Arithmetic Topics. J. Ed. Psychol. 37: 293-315; May 1946.

Morton, R. L. An Analysis of Pupils' Errors in Fractions. J. Ed. Res. 9: 117-125; Feb. 1924.

Morton, R. L. An Analysis of Errors in the Solution of Arithmetic Problems. Ed. Res. B. 4: 187-190; Apr. 29, 1923.

Morton, R. L. The Influence of Pupil Conduct on Teachers' Marks. Ed. Res. B. 11: 57-60; Feb. 3, 1932.

Morton, R. L. Estimating Quotient Figures When Dividing by Two-Place Numbers. El. Sch. J. 48: 141-148; Nov. 1947.

Moseley, Nicholas. Out-of-School Arithmetic of Sixth Grade Children. Math. Teach. 31: 363-364; Dec. 1938.

Mosher, Howard H. Subject Preferences of Girls and Boys. Sch. R. 60: 34-38; Jan. 1952.

Mott, Sina M. Number Concepts of Small Children. Math. Teach. 38: 291-301; Nov. 1945.

Mott, Sina and Martin, Mary Elizabeth. Do First Graders Retain Number Concepts Learned in Kindergarten? Math. Teach. 40: 75-78; Feb. 1947.

Mulholland, Vernie. Fifth Grade Children Discover Fractions. Sch. Sci. Math. 54: 13-30; Jan. 1954.

Murray, Frank B. Conservation of Illusion-Distorted Lengths and Areas by Primary School Children. J. Ed. Psychol. 56: 62-66; Apr. 1965.

Murray, Frank B. Cognitive Conflict and Reversibility Training in the Acquisition of Length Conservation. J. Ed. Psychol. 59: 82-87; Apr. 1968.

Muscia, Robert D. Factors Related to Quantitative Understanding in the Sixth Grade. Arith. Teach. 9: 258-262; May 1962.

Myers, G. C. The Price of Speed Pressure in the Learning of Number. Ed. Res. B. 7: 265-268; Sept. 19, 1928.

Myers, Garry C. and Myers, Caroline E. Finding Mistakes Versus Correct Associations in Simple Number-Learning. J. Ed. Res. 18: 25-31; June 1928.

Myers, Garry C. and Myers, Caroline C. The Cost of Quick Shifting in Number Learning. Ed. Res. B. 7: 327-334; Oct. 31, 1928.

Namy, Elmer. Intellectual and Academic Characteristics of Fourth Grade Gifted and Pseudogifted Students. J. Excep. Child. 34: 15-18; Sept. 1967.

Nasca, Donald. Comparative Merits of a Manipulative Approach to Second-Grade Arithmetic. Arith. Teach. 13: 221-226; Mar. 1966.

Neal, Elma A. and Foster, Inez. An Experiment With Remedial Work in Common Fractions. El. Sch. J. 29: 280-283; Dec. 1928.

Nee, Mary McGreal. The Individual Progress Method in Arithmetic. Chicago Sch. J. 21: 111-115; Dec. 1939.

Nelson, L. Doyal. Textbook Difficulty and Mathematics Achievement in Junior High School. Math. Teach. 58: 724-729; Dec. 1965.

Nelson, L. Doyal and Worth, Walter H. Mathematical Competence of Prospective Elementary Teachers in Canada and in the United States. Arith. Teach. 8: 147-151; Apr. 1961.

Neureiter, Paul R. and Wozencraft, Marian. What Arithmetic in Second Grade? *Arith. Teach.* 9: 252-257; May 1962.

Newcomb, R. S. Teaching Pupils How to Solve Problems in Arithmetic. *El. Sch. J.* 23: 183-189; Nov. 1922.

Newcomb, R. S. Securing the Maximum Amount of Work From Every Pupil. *El. Sch. J.* 25: 376-379; Jan. 1925.

Newcomb, R. S. Effective Drill Exercises in Arithmetic. *J. Ed. Psychol.* 16: 127-131; Feb. 1925.

Newland, T. Ernest. A Study of the Specific Illegibilities Found in the Writing of Arabic Numerals. *J. Ed. Res.* 21: 177-185; Mar. 1930.

Newman, Thomas B. and Seiser, William. The Floating Teacher--Help for the Mathematically Disadvantaged. *Math. Teach.* 60: 753-755; Nov. 1967.

Nicholls, R. H. Programming Piaget in Practice. *Teach. Arith.: Br. Elem. Math. J.* 1: 24-38; Autumn 1963.

Noon, Philo G. The Child's Use of Numbers. *J. Ed. Psychol.* 10: 462-467; Nov. 1919.

O'Brien, F. P. Co-operative Experiment Pertaining to Instruction in Arithmetic. *Am. Ed.* 31: 219-221; Feb. 1928.

Ojemann, Ralph H.; Maxey, E. James; and Snider, Bill C. F. The Effect of a Program of Guided Learning Experiences in Developing Probability Concepts at the Third Grade Level. *J. Exp. Ed.* 33: 321-330; June 1965.

Olander, Clarence E. The Use of a Readiness Test in Teaching a Unit on Signed Numbers. *Sch. Sci. Math.* 57: 131-138; Feb. 1957.

Olander, Herbert T. Transfer of Learning in Simple Addition and Subtraction. I. *El. Sch. J.* 31: 358-369; Jan. 1931. II. 31: 427-437; Feb. 1931.

Olander, Herbert T. The Need for Diagnostic Testing. *El. Sch. J.* 33: 736-745; June 1933.

Olander, Herbert T. and Brown, Betty Irene. A Research in Mental Arithmetic Involving Subtraction. *J. Ed. Res.* 53: 97-102; Nov. 1959.

Olander, Herbert T. and Sharp, E. Preston. Long Division Versus Short Division. *J. Ed. Res.* 26: 6-11; Sept. 1932.

Olander, Herbert T.; Van Wagenen, M. J.; and Bishop, Helen Miriam. Predicting Arithmetic Achievement. J. Ed. Res. 43: 66-73; Sept. 1949.

Oldham, Hilda W. A Psychological Study of Mathematical Ability, With Special Reference to School Mathematics. Br. J. Ed. Psychol. 7: 269-286; Nov. 1937 (Part I); 8: 16-27; Feb. 1938 (Part II).

Orleans, Jacob S. and Sperling, Julia L. The Arithmetic Knowledge of Graduate Students. J. Ed. Res. 48: 177-186; Nov. 1954.

Orleans, Jacob S. and Wandt, Edwin. The Understanding of Arithmetic Possessed by Teachers. El. Sch. J. 53: 501-507; May 1953.

O'Rourke, Everett V. and Mead, Cyrus D. Vocabulary Difficulties of Five Textbooks in Third-Grade Arithmetic. El. Sch. J. 41: 683-691; May 1941.

Osborne, Raymond W. and Gillet, Harry O. Mental Age is Important Factor in Teaching Arithmetic. Nation's Schools 12: 19-24; July 1933.

Osburn, W. J. How Shall We Subtract? J. Ed. Res. 16: 237-246; Nov. 1927.

Osburn, Worth J. Levels of Difficulty in Long Division. El. Sch. J. 46: 441-447; Apr. 1946.

Osburn, W. J. Division by Dichotomy as Applied to the Estimation of Quotient Figures. El. Sch. J. 50: 326-330; Feb. 1950.

Osburn, W. J. and Drennan, L. J. Problem Solving in Arithmetic. Ed. Res. B. 10: 123-128; Mar. 4, 1931.

Osburn, W. J. and Foltz, P. J. Permanence of Improvement in the Fundamentals of Arithmetic. Ed. Res. B. 10: 227-234; Apr. 29, 1931.

Otto, H. J. Remedial Instruction in Arithmetic. El. Sch. J. 28: 124-133; Oct. 1927.

Otto, Wayne. Inhibitory Potential in Good and Poor Achievers. J. Ed. Psychol. 56: 200-207; Aug. 1965.

Overholt, Elbert D. A Piagetian Conservation Concept. Arith. Teach. 12: 317-326; May 1965.

Overman, J. R. An Experimental Study of the Effect of the Method of Instruction on Transfer of Training in Arithmetic. El. Sch. J. 31: 183-190; Nov. 1930.

Pace, Angela. Understanding and the Ability to Solve Problems. *Arith. Teach.* 8: 226-233; May 1961.

Pace, Angela. Understanding of Basic Concepts of Arithmetic: A Comparative Study. *J. Ed. Res.* 60: 107-120; Nov. 1966.

Pace, Angela. The Effect of Instruction Upon the Development of the Concept of Number. *J. Ed. Res.* 62: 183-189; Dec. 1968.

Paige, Donald D. Learning While Testing. *J. Ed. Res.* 59: 276-277; Feb. 1966.

Paige, Donald D. A Comparison of Team Versus Traditional Teaching of Junior High School Mathematics. *Sch. Sci. Math.* 47: 365-367; Apr. 1967.

Paige, Donald D. and Jennings, Margaret. Measurement in the Elementary School. *Arith. Teach.* 14: 354-357; May 1967.

Partridge, C. M. Number Needs in Children's Reading Activities. *El. Sch. J.* 26: 356-366; Jan. 1926.

Paschal, Billy J. A Concerned Teacher Makes the Difference. *Arith. Teach.* 13: 203-205; Mar. 1966.

Passehl, George. Teaching Arithmetic Through Activity Units. *Peabody J. Ed.* 27: 148-152; Nov. 1949.

Passy, Robert A. The Effect of Cuisenaire Materials on Reasoning and Computation. *Arith. Teach.* 10: 439-440; Nov. 1963.

Passy, Robert A. Socio-Economic Status and Mathematics Achievement. *Arith. Teach.* 11: 469-470; Nov. 1964.

Pate, Robert Thomas. Transactional Pattern Differences Between School Mathematics Programs. *Arith. Teach.* 13: 21-25; Jan. 1966.

Payne, Holland. What About Modern Programs in Mathematics? *Math. Teach.* 58: 422-424; May 1965.

Peck, Hugh I. An Evaluation of Topics in Modern Mathematics. *Arith. Teach.* 10: 277-279; May 1963.

Peel, E. A. Experimental Examination of Some of Piaget's Schemata Concerning Children's Perception and Thinking, and a Discussion of Their Educational Significance. *Br. J. Ed. Psychol.* 29: 89-103; June 1959.

Pella, Milton O. Some Observations Concerning Science and Mathematics Instruction in Three Countries of the Middle East. *Sch. Sci. Math.* 65: 729-737; Nov. 1965.

Perreault, (Sister) Mary Jacqueline. Are We Trying to Force Mathematical Maturity? *Sch. Sci. Math.* 57: 515-522; Oct. 1957.

Perrodin, Alex F. and Snipes, Walter T. The Relationship of Mobility to Achievement in Reading, Arithmetic, and Language in Selected Georgia Elementary Schools. *J. Ed. Res.* 59: 315-319; Mar. 1966.

Petty, Olan. Non-Pencil-and-Paper Solutions of Problems. *Arith. Teach.* 12: 229-235; Dec. 1965.

Phelps, C. L. A Study of Errors in Tests of Adding Ability. *El. Sch. Teach.* 14: 29-39; 1913.

Phillips, Clarence. Background and Mathematical Achievement of Elementary Education Students in Arithmetic for Teachers. *Sch. Sci. Math.* 53: 48-52; Jan. 1953.

Phillips, Clarence. The Relationship Between Arithmetic Achievement and Vocabulary Knowledge of Elementary Mathematics. *Arith. Teach.* 7: 240-242; May 1960.

Phillips, Clarence. Approach to the Training of Prospective Elementary Mathematics Teachers. *J. Teach. Ed.* 19: 293-297; Fall 1968.

Phillips, F. M. Value of Daily Drill in Arithmetic. *J. Ed. Psychol.* 4: 159-163; Mar. 1913.

Phillips, Frank McGinley. Relation of Initial Ability to the Extent of Improvement in Certain Mathematical Traits. *Pedagog. Sem.* 26: 330-355; Dec. 1919.

Phillips, John L., Jr. Perception in Number Skills - A Study in Tachistoscopic Training. *J. Ed. Psychol.* 45: 459-468; Dec. 1954.

Piaget, Jean. How Children Form Mathematical Concepts. *Scientific Am.* 189: 74-79; 1953.

Pick, Herbert L., Jr. and Pick, Anne D. A Developmental and Analytic Study of the Size-Weight Illusion. *J. Exp. Child Psychol.* 5: 362-371; Sept. 1967.

Pigge, Fred L. Analysis of Covariance in a Randomly Replicated Arithmetic Methods Experiment. *J. Exp. Ed.* 34: 73-83; Summer 1966.

Pigge, Fred L. Frequencies of Unwritten Algorisms. Arith. Teach. 14: 588-593; Nov. 1967. (see: Shuster, Albert H. and Pigge, Fred L. Retention Efficiency of Meaningful Teaching. Arith. Teach. 12: 24-32; Jan. 1965.)

Pincus, Morris. Enrichment in Arithmetic for the Primary Grades. Arith. Teach. 7: 412-413, 417; Dec. 1960.

Pinney, Grant C. Grouping by Arithmetic Ability - An Experiment in the Teaching of Arithmetic. Arith. Teach. 8: 120-123; Mar. 1961.

Pistor, Frederick. Measuring Some Subtle Values of Progressive Education. Ed. Meth. 14: 118-125; Dec. 1934.

Pitts, Vera L. An Investigation of the Relationships Between Two Preschool Programs on the Adjustment and Readiness of Disadvantaged Pupils. Childhd. Ed. 44: 524-525; Apr. 1968.

Plank, Emma N. Observations on Attitudes of Young Children Toward Mathematics. Math. Teach. 43: 252-263; Oct. 1950.

Polkinghorne, Ada R. Young Children and Fractions. Childhd. Ed. 11: 354-358; May 1935.

Porterfield, O. V. Ambiguities in Teaching Arithmetic. Arith. Teach. 12: 348-351; May 1965.

Potamkin, Caroline C. An Experiment in Individualized Arithmetic. El. Sch. J. 64: 155-162; Dec. 1963.

Potter, Mary C. and Levy, Ellen I. Spatial Enumeration Without Counting. Child Develop. 39: 265-272; Mar. 1968.

Powell, Lucile B. A Spark of Enthusiasm. Arith. Teach. 8: 239-241; May 1961.

Powell, Marvin; O'Connor, Henry A.; Deutsch, Murray; and Parsley, Kenneth M., Jr. Are There Really Sex Differences in Achievement? J. Ed. Res. 57: 210-212; Dec. 1963.

Powell, Marvin; O'Connor, Henry A.; and Parsley, Kenneth M., Jr. Further Investigation of Sex Differences in Achievement of Under-, Average-, and Over-Achieving Students Within Five I.Q. Groups in Grades Four Through Eight. J. Ed. Res. 57: 268-270; Jan. 1964.

Pratoomraj, Sawat and Johnson, Ronald C. Kinds of Questions and Types of Conservation Tasks as Related to Children's Conservation Responses. Child Develop. 37: 343-353; June 1966.

Pressey, L. C. and Elam, M. K. The Fundamental Vocabulary of Elementary-School Arithmetic. *El. Sch. J.* 33: 46-50; Sept. 1932.

Pressey, L. C. and Moore, W. S. The Growth of Mathematical Vocabulary From the Third Grade Through High School. *Sch. R.* 40: 449-454; June 1932.

Price, Edette B.; Prescott, Arthur L.; and Hopkins, Kenneth D. Comparative Achievement With Departmentalized and Self-Contained Classroom Organization. *Arith. Teach.* 14: 212-215; Mar. 1967.

Priore, Angela. Achievement by Pupils Entering First Grade. *Arith. Teach.* 4: 55-60; Mar. 1957.

Pritchard, John W. Motor Performance as a Chance Factor in Test Scores. *J. Ed. Res.* 37: 181-192; Nov. 1943.

Prouse, Howard L. Creativity in School Mathematics. *Math. Teach.* 14: 876-879; Dec. 1967.

Provus, Malcolm M. Ability Grouping in Arithmetic. *El. Sch. J.* 60: 391-398; Apr. 1960.

Pusey, Mrs. Harriet C. Arithmetic Achievement of Japanese-Americans. *Math. Teach.* 38: 172-174; Apr. 1945.

Pyle, W. H. An Experimental Study of the Development of Certain Aspects of Reasoning. *J. Ed. Psychol.* 26: 539-546; Oct. 1935.

Rainey, Dan S. and Kelley, Francis J. An Evaluation of a Programed Textbook With Educable Mentally Retarded Children. *J. Excep. Child.* 34: 125-126; Oct. 1967.

Ramharter, Hazel K. and Johnson, Harry C. Methods of Attack Used by "Good" and "Poor" Achievers in Attempting to Correct Errors in Six Types of Subtraction Involving Fractions. *J. Ed. Res.* 42: 586-597; Apr. 1949.

Randall, Joseph H. Corrective Arithmetic in Junior High School. *Ed. Meth.* 16: 182-185; Jan. 1937.

Randolph, Paul H. An Experiment in Programmed Instruction in Junior High School. *Math. Teach.* 57: 160-162; Mar. 1964.

Rappaport, David. Understanding Meanings in Arithmetic. *Arith. Teach.* 5: 96-99; Mar. 1958.

Rappaport, David. Preparation of Teachers of Arithmetic. *Sch. Sci. Math.* 58: 636-643; Nov. 1958.

Rappaport, David. The Meanings Approach in Teaching Arithmetic. Chicago Sch. J. 44: 172-174; Jan. 1963.

Raths, Louis E. Grade Placement of Addition and Subtraction of Fractions. Ed. Res. B. 11: 29-38; Jan. 20, 1932.

Rausch, Oscar P. The Retention by Teachers of Computational Skills in Arithmetic. Math. Teach. 40: 178-179; Apr. 1947.

Reavis, W. C. The Social Motive in the Teaching of Arithmetic. El. Sch. J. 18: 264-267; Dec. 1917.

Redbird, Helen. Individualizing Arithmetic Instruction. Arith. Teach. 11: 348-349; May 1964.

Reddell, William D. and DeVault, M. Vere. In-Service Research in Arithmetic Teaching Aids. Arith. Teach. 7: 243-246; May 1960.

Reese, Hayne W. Manifest Anxiety and Achievement Test Performance. J. Ed. Psychol. 52: 132-135; June 1961.

Reid, Florence E. Incidental Number Situations in First Grade. J. Ed. Res. 30: 36-43; Sept. 1936.

Renwick, E. M. Children's Misconceptions Concerning the Symbols for Mathematical Equality. Br. J. Ed. Psychol. 2: 173-183; June 1932.

Repp, Florence C. The Vocabularies of Five Recent Third Grade Arithmetic Textbooks. Arith. Teach. 7: 128-132; Mar. 1960.

Retzer, Kenneth A. and Henderson, Kenneth B. Effect of Teaching Concepts of Logic on Verbalization of Discovered Mathematical Generalizations. Math. Teach. 40: 707-710; Nov. 1967.

Reys, Robert E. Are Elementary School Teachers Satisfied With Their Mathematics Preparation? Arith. Teach. 14: 190-193; Mar. 1967.

Reys, Robert. Mathematical Competencies of Elementary Education Majors. J. Ed. Res. 61: 265-266; Feb. 1968.

Reys, Robert E. Mathematical Competencies of Preservice Elementary School Teachers. Sch. Sci. Math. 68: 302-308; Apr. 1968.

Reys, Robert and Delon, Floyd G. Attitudes of Prospective Elementary School Teachers Towards Arithmetic. Arith. Teach. 15: 364-366; Apr. 1968.

Reys, Robert E. and Knowles, Lois. What is the Status of Elementary School Mathematics? El. Sch. J. 68: 167-171; Jan. 1968.

Rheins, Gladys B. and Rheins, Joel J. A Comparison of Two Methods of Compound Subtraction. *Arith. Teach.* 2: 63-69; Oct. 1955.

Rice, Joseph P., Jr. A Comparative Study of Academic Interest Patterns Among Selected Groups of Exceptional and Normal Intermediate Children. *Calif. J. Ed. Res.* 14: 131-137, 144; May 1963.

Richardson, Helen M. and Surko, Elise. WISC Scores and Status in Reading and Arithmetic of Delinquent Children. *J. Genet. Psychol.* 89: 251-262; Oct. 1956.

Richardson, J. W. The Campaign Method in Elementary Education. *J. Ed. Res.* 2: 481-492; June 1920.

Richtmeyer, Cleon C. Functional Mathematical Needs of Teachers. *J. Exp. Ed.* 6: 396-398; June 1938.

Ridding, L. W. An Investigation of Personality Measures Associated With Over and Under Achievement in English and Arithmetic. *Br. J. Ed. Psychol.* 37: 397-398; Nov. 1967.

Riedesel, Alan. Why Teach Bank Discount? *Arith. Teach.* 4: 268; Dec. 1957.

Riedesel, C. Alan. Verbal Problem Solving: Suggestions for Improving Instruction. *Arith. Teach.* 11: 312-316; May 1964.

Riedesel, C. Alan and Suydam, Marilyn N. Computer-Assisted Instruction: Implications for Teacher Education. *Arith. Teach.* 14: 24-29; Jan. 1967.

Rimoldi, H. J. A.; Aghi, M.; and Burger, G. Some Effects of Logical Structure, Language, and Age in Problem Solving in Children. *J. Genet. Psychol.* 112: 127-143; Mar. 1968.

Risden, Gladys A. Learning for Remembering. *Ed. Meth.* 20: 65-69; Nov. 1940.

Risden, Gladys. A Remedy is Suggested for Math Carelessness. *The Clearing House* 31: 203-206; Dec. 1956.

Roantree, William F. The Question of Method in Subtraction. *Math. Teach.* 17: 110-116; 1924.

Roberts, Dorothy M. and Bloom, Irving. Mathematics in Kindergarten--Formal or Informal? *El. Sch. J.* 67: 338-341; Mar. 1967.

Roberts, Gerhard H. The Failure Strategies of Third Grade Arithmetic Pupils. *Arith. Teach.* 15: 442-446; May 1968.

Robinson, Arthur E. Are We Teaching Arithmetic Effectively? A Summary of a Recent Study. *Math. Teach.* 28: 215-222; Apr. 1935.

Robinson, Halbert B. An Experimental Examination of the Size-Weight Illusion in Young Children. *Child Develop.* 35: 91-107; Mar. 1964.

Robertson, M. S. Problem Solving in Arithmetic. *Peabody J. Ed.* 9: 176-183; Nov. 1931.

Rolker, Edna. The Spread of Ability in Arithmetic. *Baltimore B. Ed.* 9: 5-7; Sept. 1930.

Rolston, Dorothy M. and Spitzer, Herbert F. Oral and Written Expression of Numbers of Three or More Digits. *El. Sch. J.* 42: 116-119; Oct. 1941.

Romberg, Thomas A. A Note on Multiplying Fractions. *Arith. Teach.* 15: 263-265; Mar. 1968.

Romberg, Thomas A. and Wilson, James W. The Development of Mathematics Achievement Tests for the National Longitudinal Study of Mathematical Abilities. *Math. Teach.* 61: 489-495; May 1968.

Rose, Alvin and Rose, Helen. Intelligence, Sibling Position, and Sociocultural Background as Factors in Arithmetic Performance. *Arith. Teach.* 8: 50-56; Feb. 1961.

Ross, Ramon. A Description of Twenty Arithmetic Underachievers. *Arith. Teach.* 11: 235-241; Apr. 1964.

Rosse, James C. An Experiment to Test the Increase in Reasoning Ability From the Use of Test and Practice Sheets in 6A Arithmetic. *J. Ed. Res.* 22: 210-213; Oct. 1930.

Rossman, John G. Problems in Non-Promotion. *Ind. U. Sch. Ed. Bull.* 3: 47-58; Apr. 1927.

Rowland, Monroe and Inskeep, James. Subject Preferences of Upper Elementary School Children in Cajon Valley Union School District. *Calif. J. Ed. Res.* 14: 187-191; Sept. 1963.

Ruch, G. M. Relative Difficulty of the One Hundred Multiplication Facts With Special Reference to Textbook Construction. *El. Sch. J.* 32: 369-377; Jan. 1932.

Ruddell, Arden K. The Results of a Modern Mathematics Program. *Arith. Teach.* 9: 330-335; Oct. 1962.

Ruddell, Arden K. and Balow, Irving H. Evaluation of an In-Service Arithmetic Program. J. Ed. Res. 57: 34-38; Sept. 1963.

Ruddell, Arden K. and Brown, Gerald W. In-Service Education in Arithmetic: Three Approaches. El. Sch. J. 64: 377-382; Apr. 1964.

Rusch, Reuben R.; Brown, John A.; and Delong, Arthur R. Meaning of an Arithmetic Test Score. Arith. Teach. 9: 145-148; Mar. 1962.

Rusch, Reuben R. and Clark, Richard M. Four Years in Three: An Evaluation. El. Sch. J. 64: 281-285; Feb. 1963.

Russell, Ned M. Arithmetical Concepts of Children. J. Ed. Res. 29: 647-663; May 1936.

Sassenrath, Julius M. and Welch, Ronald C. Teacher Preparation and Teaching Tasks. Calif. J. Ed. Res. 19: 112-120; May 1968.

Sato, Ryoichiro. Commentary on the International Study of Achievement in Mathematics. Arith. Teach. 15: 103-107; Feb. 1968.

Satterly, David. Perceptual, Representational and Conceptual Characteristics of Primary School Children. Br. J. Ed. Psychol. 38: 78-82; Feb. 1968.

Savard, William G. An Evaluation of an Ability Grouping Program. Calif. J. Ed. Res. 11: 56-60; Mar. 1960.

Sawada, Daiyo and Nelson, L. Doyal. Conservation of Length and the Teaching of Linear Measurement: A Methodological Critique. Arith. Teach. 14: 345-348; May 1967.

Sax, Gilbert and Ottina, John R. The Arithmetic Achievement of Pupils Differing in School Experience. Calif. J. Ed. Res. 9: 15-19; Jan. 1958.

Scandura, Joseph M. An Analysis of Exposition and Discovery Modes of Problem Solving Instruction. J. Exp. Ed. 33: 148-159; Dec. 1964.

Scaramuzzi, Louis E. Money is Only Imaginary. The Clearing House 30: 280-283; Jan. 1956.

Schane, Evelyn Bessie. Characteristic Errors in Common Fractions at Different Levels of Intelligence. Pittsburgh Sch. 12: 155-168; Mar. 1938.

Scheidler, Dorothy and McFadden, Elsie. Arithmetic - A Road to Happiness. Ohio Sch. 21: 62-63; Feb. 1943.

Schell, Leo M. Learning the Distributive Property by Third Graders. Sch. Sci. Math. 68: 28-32; Jan. 1968.

Schell, Leo M. and Burns, Paul C. Pupil Performance With Three Types of Subtraction Situations. Sch. Sci. Math. 62: 208-214; Mar. 1962.

Schlinsog, George W. The Effects of Supplementing Sixth-Grade Instruction With a Study of Nondecimal Numbers. Arith. Teach. 15: 254-263; Mar. 1968.

Schmitt, Clara. Extreme Retardation in Arithmetic. El. Sch. J. 21: 528-547; Mar. 1921.

Schoonover, Sarah M. A Longitudinal Study of Sibling Resemblance in Intelligence and Achievement. J. Ed. Psychol. 47: 436-442; Nov. 1956.

Schorling, Raleigh. The Need for Being Definite With Respect to Achievement Standards. Math. Teach. 24: 311-329; May 1931.

Schott, Andrew F. New Tools, Methods for Their Use, and a New Curriculum in Arithmetic. Arith. Teach. 4: 204-209; Nov. 1957.

Schutter, Charles H. Some Important Features of European Arithmetic Programs. Arith. Teach. 7: 1-8; Jan. 1960.

Scott, Carrie M. The Relationship Between Intelligence Quotients and Gain in Reading Achievement With Arithmetic Reasoning, Social Studies, and Science. J. Ed. Res. 56: 322-326; Feb. 1963.

Scott, Carrie M. Intelligence and Gain in Reading as Related to Gains in the Sub-tests of the Stanford Achievement Test. J. Ed. Res. 56: 494-496; May/June 1963.

Scott, Lloyd. Children's Concept of Scale and the Subtraction of Fractions. Arith. Teach. 9: 115-118; Mar. 1962.

Scott, Lloyd. A Teaching Investigation of the Introduction of Various Radices Into Kindergarten and First Grade Mathematics. Calif. J. Ed. Res. 14: 3-10; Jan. 1963.

Scott, Lloyd. A Study of Teaching Division Through the Use of Two Algorisms. Sch. Sci. Math. 63: 739-752; Dec. 1963.

Scott, Lloyd. Children's Perception of Mathematical Inconsistencies. Arith. Teach. 12: 617-624; Dec. 1965.

Scott, Lloyd. A Study of the Case for Measurement in Elementary School Mathematics. Sch. Sci. Math. 66: 714-722; Nov. 1966.

Scott, Lloyd F. Summer Loss in Modern and Traditional Elementary School Mathematics Programs. *Calif. J. Ed. Res.* 28: 145-151; May 1967.

Scott, Ralph and Lighthall, Frederick F. Relationship Between Content, Sex, Grade, and Degree of Disadvantage in Arithmetic Problem Solving. *J. Sch. Psychol.* 6: 61-67; Fall 1967.

Scrivner, A. W. and Urbanek, R. The Value of "Teacher-Aide" Participation in the Elementary School. *Arith. Teach.* 10: 84-87; Feb. 1963.

Semmelmeyer, Madeline. Extensional Methods in Dealing With Abstractions in Reading. *El. Sch. J.* 50: 28-36; Sept. 1949.

Sganga, Frank. An Experiment With Individualized Arithmetic. *Instruc.* 69: 52, 88; Feb. 1960.

Shantz, Carolyn Uhlinger and Smock, Charles D. Development of Distance Conservation and the Spatial Coordinate System. *Child Develop.* 37: 943-948; Dec. 1966.

Sharples, Aneita; Sutton-Smith, B.; Exner, J.; and Rosenberg, B. G. Logical Analysis and Transitivity. *J. Genet. Psychol.* 112: 21-25; Mar. 1968.

Shepard, Winifred O. and Schaeffer, Maurice. The Effect of Concept Knowledge on Discrimination Learning. *Child Develop.* 27: 173-178; June 1956.

Sherman, Homer C. and Belding, Robert E. Are Soviet Arithmetic Books Better Than Ours? *Arith. Teach.* 12: 633-637; Dec. 1965.

Shim, Chung-Phing. A Study of the Cumulative Effect of Four Teacher Characteristics on the Achievement of Elementary School Pupils. *J. Ed. Res.* 59: 33-34; Sept. 1965.

Shine, Aileen. Relationship Between Arithmetic Achievement and Item Performance on the Revised Stanford-Binet Scale. *Arith. Teach.* 8: 57-59; Feb. 1961.

Shipp, Donald E. and Deer, George H. The Use of Class Time in Arithmetic. *Arith. Teach.* 7: 117-121; Mar. 1960.

Shryock, Jerry. A Mathematics Course for Prospective Elementary School Teachers. *Arith. Teach.* 10: 208-211; Apr. 1963.

Shuster, Albert and Pigge, Fred. Retention Efficiency of Meaningful Teaching. *Arith. Teach.* 12: 24-31; Jan. 1965.

Silverman, Irwin and Schneider, Dale S. A Study of the Development of Conservation by a Nonverbal Method. J. Genet. Psychol. 112: 287-291; June 1968.

Sitts, Elizabeth and Sitts, Marvin. Parents and Children Learn About Arithmetic. El. Sch. J. 64: 31-35; Oct. 1963.

Skapski, Mary King. Ungraded Primary Reading Program: An Objective Evaluation. El. Sch. J. 61: 41-45; Oct. 1960.

Skemp, Richard R. Reflective Intelligence and Mathematics. Br. J. Ed. Psychol. 31: 45-55; Feb. 1960.

Skypek, Dora Helen. A Comparison of the Mathematical Competencies of Education and Non-Education Majors Enrolled in a Liberal Arts College. Am. Math. Mon. 72: 770-773; Sept. 1965.

Sloan, Fred A., Jr. and Pate, Robert Thomas. Teacher-Pupil Interaction in Two Approaches to Mathematics. El. Sch. J. 67: 161-167; Dec. 1966.

Smedslund, Jan. The Acquisition of Transitivity of Weight in Five-to-Seven-Year-Old Children. J. Genet. Psychol. 102: 245-255; Apr. 1963.

Smedslund, Jan. Development of Concrete Transitivity of Lengths in Children. Child Develop. 34: 389-405; June 1963.

Smith, E. E. and Burton, W. H. Effect Upon Pupil Achievement in Arithmetic Reasoning and Arithmetic Computation of a Supervisory Program Emphasizing Computation. Ed. Meth. 12: 267-271; Feb. 1933.

Smith, Frank. Prospective Teachers' Attitudes Toward Arithmetic. Arith. Teach. 11: 474-477; Nov. 1964.

Smith, Frank. How Well are Colleges Preparing Teachers for Modern Mathematics?--An Answer. Arith. Teach. 14: 200-202; Mar. 1967.

Smith, Henry Lester and Eaton, Merrill Thomas. The Relation of Accuracy to Speed in Addition. Ind. U. Sch. Ed. Bull. 14: 5-23; 1938.

Smith, Henry Lester and Eaton, Merrill Thomas. A Diagnostic Study of Efficiency in Arithmetic. Ind. U. Sch. Ed. Bull. 15: 3-49; 1939.

Smith, Henry Lester and Eaton, Merrill Thomas. An Analysis of Arithmetic Textbooks (First Period - 1790 to 1820). Ind. U. Sch. Ed. Bull. 18: 1-52; Jan. 1942.

Smith, Henry Lester and Eaton, Merrill Thomas. An Analysis of Arithmetic Textbooks (Second Period - 1821 to 1850). *Ind. U. Sch. Ed. Bull.* 18: 1-108; Nov. 1942.

Smith, Henry Lester and Eaton, Merrill Thomas. An Analysis of Arithmetic Textbooks (Third Period - 1851 to 1880). *Ind. U. Sch. Ed. Bull.* 18: 1-108; Nov. 1942.

Smith, Henry Lester and Eaton, Merrill Thomas. An Analysis of Arithmetic Textbooks (Fourth Period - 1881 to 1910). *Ind. U. Sch. Ed. Bull.* 19: 1-58; July 1943.

Smith, Henry Lester and Eaton, Merrill Thomas. An Analysis of Arithmetic Textbooks (Fifth Period - 1911 to 1940). *Ind. U. Sch. Ed. Bull.* 19: 1-41; Nov. 1943.

Smith, Henry Lester and others. One Hundred Fifty Years of Arithmetic Textbooks. *Ind. U. Sch. Ed. Bull.* 21: 1-149; Jan. 1945.

Smith, Ian D. The Effects of Training Procedures Upon the Acquisition of Conservation of Weight. *Child Develop.* 39: 515-526; June 1968.

Smith, James H. Individual Variations in Arithmetic. *El. Sch. J.* 17: 195-200; Nov. 1916.

Smith, J. H. Arithmetical Combinations. *El. Sch. J.* 21: 762-770; June 1921.

Smith, Kenneth J. and Heddens, James W. The Readability of Experimental Mathematics Materials. *Arith. Teach.* 11: 391-394; Oct. 1964.

Smith, Lehi T. Curricula for Education of Teachers. *Am. Math. Mon.* 70: 202-203; Feb. 1963.

Smith, Nila B. An Investigation of the Uses of Arithmetic in the Out-of-School Life of First-Grade Children. *El. Sch. J.* 24: 621-626; Apr. 1924.

Snipes, Walter T. Mobility on Arithmetic Achievement. *Arith. Teach.* 13: 43-46; Jan. 1966.

Soth, M. R. A Study of a Pupil Retarded in Arithmetic. *El. Sch. J.* 29: 439-442; Feb. 1929.

Souder, Hugh Cowan. The Construction and Evaluation of Certain Readiness Tests in Common Fractions. *J. Ed. Res.* 37: 127-134; Oct. 1943.

Spagnoli, Joseph. An Experience With Programmed Materials. *J. Ed. Res.* 58: 447-448; July/Aug. 1965.

Sparks, Jack N. Arithmetic Understandings Needed by Elementary School Teachers. *Arith. Teach.* 8: 395-403; Dec. 1961.

Spaulding, F. T. An Analysis of the Content of Six Third-Grade Arithmetics. *J. Ed. Res.* 4: 413-423; Dec. 1921.

Spayde, Paul E. Kindergarten Children's Familiarity With Measurement. *Ed. Res. B.* 32: 234-238; Dec. 9, 1953.

Spiker, Charles C. and Terrell, Glenn, Jr. Factors Associated With Transposition Behavior of Preschool Children. *J. Genet. Psychol.* 36: 143-158; Jan. 1955.

Spitzer, Herbert F. and Flournoy, Frances. Developing Facility in Solving Verbal Problems. *Arith. Teach.* 3: 177-182; Nov. 1956.

Springer, Doris V. Development of Concepts Related to the Clock as Shown in Young Children's Drawings. *J. Genet. Psychol.* 79: 47-54; 1951.

Springer, Doris. Development in Young Children of an Understanding of Time and the Clock. *J. Genet. Psychol.* 80: 83-96; 1952.

Springer, Isidore. Teaching Denominate Numbers. *J. Ed. Psychol.* 6: 630-632; Dec. 1915.

Stadtlander, Elizabeth. Arithmetic Theories, A State Course of Study, and Textbooks. *El. Sch. J.* 41: 438-453; Feb. 1941.

Steffe, Leslie P. The Relationship of Conservation of Numerousness to Problem-Solving Abilities of First-Grade Children. *Arith. Teach.* 15: 47-52; Jan. 1968.

Steiner, M. A. Value of Home-Study Assignments. *Sch. and Soc.* 40: 20-24; July 7, 1934.

Steinway, Louise S. An Experiment in Games Involving a Knowledge of Number. *Teach. Col. Rec.* 19: 43-53; Jan. 1918.

Stendler, Celia Burns. Social Class Differences in Parental Attitude Toward School at Grade 1 Level. *Child Develop.* 22: 37-46; Mar. 1951.

Stephens, J. M. and Lichtenstein, Arthur. Factors Associated With Success in Teaching Grade Five Arithmetic. *J. Ed. Res.* 40: 683-694; May 1947.

Stephens, Lois. Comparison of Attitudes and Achievement Among Junior High School Mathematics Classes. Arith. Teach. 7: 351-356; Nov. 1960.

Stephens, Lois and Dutton, Wilbur. Retention of the Skill of Division of Fractions. Arith. Teach. 7: 28-31; Jan. 1960.

Stern, Carolyn. Acquisition of Problem-Solving Strategies in Young Children and Its Relation to Verbalization. J. Ed. Psychol. 58: 245-252; Apr. 1967.

Stern, Carolyn and Keislar, Evan R. Acquisition of Problem Solving Strategies by Young Children, and Its Relation to Mental Age. Am. Ed. Res. J. 4: 1-12; Jan. 1967.

Stevens, B. A. Problem Solving in Arithmetic. J. Ed. Res. 25: 253-260; Apr./May 1932.

Stevenson, Harold W. and Bitterman, M. E. The Distance-Effect in the Transposition of Intermediate Size by Children. Am. J. Psychol. 68: 274-279; June 1955.

Stevenson, P. R. Increasing the Ability of Pupils to Solve Arithmetic Problems. Ed. Res. B. 3: 267-270; Oct. 15, 1924.

Stevenson, P. R. Difficulties in Problem Solving. J. Ed. Res. 11: 95-103; Feb. 1925.

Stipanowich, Joseph. The Mathematical Training of Prospective Elementary-School Teachers. Arith. Teach. 4: 240-248; Dec. 1957.

St. John, Charles W. Some Evidences of Effects of the Pupil's Classroom Adjustment Upon His Achievement Test Performance. J. Ed. Psychol. 23: 489-504; Oct. 1932.

Stokes, C. N. Comparing the Effect of Arithmetic and General Mathematics Training in the Seventh and Eighth Grades Upon Achievement in Ninth-Grade General Mathematics. Sch. Sci. Math. 21: 853-857; Oct. 1931.

Stokes, C. Newton. 80,000 Children's Reactions to Meanings in Arithmetic. Arith. Teach. 5: 281-286; Dec. 1958.

Stokes, C. N. and Finch, F. H. A Comparison of Norms on Certain Standardized Tests in Arithmetic. El. Sch. J. 32: 785-787; June 1932.

Stone, Beth F. and Rawley, Vinton N. Educational Disability in Emotionally Disturbed Children. J. Excep. Child. 30: 423-426; May 1964.

Stoneking, Lewis W. and Welch, Ronald C. Teachers' and Students' Understanding of Arithmetic. Ind. U. Sch. Ed. Bull. 37: 1-56; Sept. 1961.

Stotlar, Carolyn. Arithmetic Concepts of Preschool Children. El. Sch. J. 46: 342-345; Feb. 1946.

Stright, Virginia M. A Study of the Attitudes Toward Arithmetic of Students and Teachers in the Third, Fourth, and Sixth Grades. Arith. Teach. 7: 280-286; Oct. 1960.

Stroud, J. B. and Lindquist, E. F. Sex Differences in Achievement in the Elementary and Secondary Schools. J. Ed. Psychol. 33: 657-667; Dec. 1942.

Stutler, Mary S. Arithmetic Concepts in First Grade. Arith. Teach. 9: 81-85; Feb. 1962.

Sueltz, Ben A. Measuring the Newer Aspects of Functional Arithmetic. El. Sch. J. 47: 323-330; Feb. 1947.

Sueltz, Ben A. Mathematical Understanding and Judgments Retained by College Freshmen. Math. Teach. 44: 13-19; Jan. 1951.

Suppes, Patrick. Mathematical Logic for the Schools. Arith. Teach. 9: 396-399; Nov. 1962.

Suppes, Patrick. Modern Learning Theory and the Elementary School Curriculum. Am. Ed. Res. J. 1: 79-93; Mar. 1964.

Suppes, Patrick. The Ability of Elementary School Children to Learn the New Mathematics. Theory into Prac. 3: 57-61; Apr. 1964.

Suppes, Patrick. Mathematical Concept Formation in Children. Am. Psychol. 21: 139-150; Feb. 1966.

Suppes, Patrick. Accelerated Program in Elementary-School Mathematics - The Second Year. Psy. in the Schools 3: 294-307; Oct. 1966.

Suppes, Patrick. Some Theoretical Models for Mathematics Learning. J. Res. and Dev. in Ed. 1: 5-22; Fall 1967.

Suppes, Patrick and Binford, Frederick. Experimental Teaching of Mathematical Logic in the Elementary School. Arith. Teach. 12: 187-195; May 1965.

Suppes, Patrick and Ihrke, Constance. Accelerated Program in Elementary-School Mathematics - The Third Year. *Psychol. in the Sch.* 4: 293-309; Sept. 1967.

Suppes, Patrick; Jerman, Max; and Groen, Guy. Arithmetic Drills and Review on a Computer-Based Teletype. *Arith. Teach.* 13: 303-309; Apr. 1966.

Suppes, Patrick and McKnight, Blair A. Sets and Numbers in Grade One, 1959-60. *Arith. Teach.* 8: 287-290; Oct. 1961.

Sutherland, John. An Investigation Into Some Aspects of Problem Solving in Arithmetic. *Br. J. Ed. Psychol.* 11: 215-222; Nov. 1941 (Part I); 12: 35-46; Feb. 1942 (Part II).

Sweet, Raymond and Dunn-Rankin, Peter. An Experiment in Team Teaching Seventh Grade Arithmetic. *Sch. Sci. Math.* 62: 341-344; May 1962.

Swenson, Esther J. Difficulty Ratings of Addition Facts as Related to Learning Method. *J. Ed. Res.* 38: 81-85; Oct. 1944.

Swineford, Frances. A Number Factor. *J. Ed. Psychol.* 40: 157-167; Mar. 1949.

Tajfel, Henri and Winter, David G. The Interdependence of Size, Number, and Value in Young Children's Estimates of Magnitude. *J. Genet. Psychol.* 102: 115-124; Jan. 1963.

Tamkin, Arthur S. A Survey of Educational Disability in Emotionally Disturbed Children. *J. Ed. Res.* 53: 313-315; Apr. 1960.

Tamkin, Arthur S. A Survey of Educational Disability in Emotionally Disturbed Children. *J. Ed. Res.* 54: 67-69; Oct. 1960.

Taylor, E. H. The Preparation of Teachers of Arithmetic in Teachers Colleges. *Math. Teach.* 30: 10-14; Jan. 1937.

Taylor, E. H. Mathematics for a Four-Year Course for Teachers in the Elementary School. *Sch. Sci. Math.* 38: 499-503; May 1938.

Taylor, Joseph S. Omitting Arithmetic in the First Year. *Ed. Adm. and Sup.* 2: 87-93; Feb. 1916.

Taylor, Joseph S. Subtraction by the Addition Process. *El. Sch. J.* 20: 203-207; Nov. 1919.

Teahan, E. Gladys. Required Home Study is Unwise. *Sch. Bd. J.* 91: 41; Nov. 1935.

Ter Keurst, Arthur J. Rote Versus Discovery Learning. *Sch. and Commun.* 55: 42, 44; Nov. 1968.

Terry, Paul Washington. How Numerals Are Read: An Experimental Study of the Reading of Isolated Numerals and Numerals in Arithmetic Problems. *Suppl. Ed. Monog.* 18: 1-110; 1922.

Thies, L. J. A Time Factor in Arithmetic Texts. *U. Iowa Monog. in Ed.* 2: 1-38; Feb. 1926.

Thompson, Gladys M. and Perrodin, Alex F. A Comparison of Arithmetic Achievement in England, Central California, and Georgia. *Arith. Teach.* 11: 181-185; Mar. 1964.

Thompson, R. B. Diagnosis and Remedial Instruction in Mathematics. *Sch. Sci. Math.* 41: 125-128; Feb. 1941.

Thompson, William Hertzog. An Experiment With the Dalton Plan. *J. Ed. Res.* 26: 493-500; Mar. 1933.

Thorndike, E. L. The Psychology of Drill in Arithmetic: The Amount of Practice. *J. Ed. Psychol.* 12: 183-194; Apr. 1921.

Thrum, Martha E. The Development of Concepts of Magnitude. *Child Develop.* 6: 120-140; June 1935.

Thurlow, Virginia. Mathematical Understanding of Seventh- and Eighth-Grade Pupils, 1948 and 1963. *Arith. Teach.* 12: 43-44; Jan. 1965.

Tilton, J. W. Individualized and Meaningful Instruction in Arithmetic. *J. Ed. Psychol.* 38: 83-88; Feb. 1947.

Tilton, J. W. An Experimental Effort to Change the Achievement Test Profile. *J. Exp. Ed.* 15: 318-323; June 1947.

Todd, Robert M. A Mathematics Course for Elementary Teachers: Does It Improve Understanding and Attitude? *Arith. Teach.* 13: 198-202; Mar. 1966.

Townsend, Robert C. An Experiment in Arithmetic Acceleration. *Arith. Teach.* 7: 409-411; Dec. 1960.

Tracy, Neal H. A Comparison of Test Results - North Carolina, California, and England. *Arith. Teach.* 6: 199-202; Oct. 1959.

Traub, Ross E. Importance of Problem Heterogeneity to Programed Instruction. *J. Ed. Psychol.* 57: 54-60; Jan. 1966.

Traweek, Melvin W. The Relationship Between Certain Personality Variables and Achievement Through Programmed Instruction. Calif. J. Ed. Res. 15: 215-220; Nov. 1964.

Traxler, Arthur E. and Selover, Margaret S. Relationship of Elementary-School Achievement Tests to Achievement Tests Taken in the Secondary School. J. Ed. Res. 36: 161-167; Nov. 1942.

Treacy, John P. The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems. J. Ed. Res. 38: 86-96; Oct. 1944.

Tredway, Daniel C. and Hollister, George E. An Experimental Study of Two Approaches to Teaching Percentage. Arith. Teach. 10: 491-495; Dec. 1963.

Triggs, E. The Value of a Desk Calculating Machine in Primary School Maths. Ed. Res. 9: 71-73; Nov. 1966.

Trousdale, Mattie S. Remedial Cases in Arithmetic, Case 3. Peabody J. Ed. 7: 290-298; Mar. 1930.

Tsao, Fei. A Study of the Relationship Between Grade and Age and Variability. J. Exp. Ed. 12: 187-200; Mar. 1944.

Turner, Richard L. and Fattu, Nicholas A. Skill in Teaching, Assessed on the Criterion of Problem Solving. Ind. U. Sch. Ed. Bull. 37: 1-30; May 1961.

Turner, Richard L. and others. Skill in Teaching, Assessed on the Criterion of Problem Solving: Three Studies. Ind. U. Sch. Ed. Bull. 39: 1-30; Jan. 1963.

Ullrich, Anna. Labeling Answers to Arithmetic Problems. Arith. Teach. 2: 148-153; Dec. 1955.

Ulrich, Louis E., Sr. 100% Automatic Response? Arith. Teach. 4: 161-167; Oct. 1957.

Ulrich, Louis E., Sr. Division by a Two-Figure Divisor. Arith. Teach. 5: 204-210; Oct. 1958.

Ulrich, Robert P. Grade Placement of Computational Topics in Arithmetic. El. Sch. J. 42: 50-59; Sept. 1941.

Unkel, Esther. A Study of the Interaction of Socioeconomic Groups and Sex Factors With the Discrepancy Between Anticipated Achievement and Actual Achievement in Elementary School Mathematics. Arith. Teach. 13: 662-670; Dec. 1966.

Uzgiris, Ina C. Situational Generality of Conservation. Child Develop. 35: 831-841; Sept. 1964.

Van Brock, Robert. Measuring Arithmetic Objectives. Arith. Teach. 12: 537-542; Nov. 1965.

VanderLinde, Louis F. Does the Study of Quantitative Vocabulary Improve Problem-Solving? El. Sch. J. 65: 143-152; Dec. 1964.

Van Engen, Lela G. Measuring Arithmetical Achievement. Math. Teach. 37: 217-218; May 1944.

Vincent, H. D. An Experimental Test of the Value of Homework in Grades Five and Six. Na. El. Prin. 16: 199-203; June 1937.

Vogeli, Bruce R. Recent Revisions in Soviet Mathematics Education. Sch. Sci. Math. 60: 425-438; June 1960.

Waggoner, Wilbur. Improving the Mathematical Competency of Teachers in Training. Arith. Teach. 5: 84-86; Mar. 1958.

Wahlstrom, Ebba L. The Computational Arithmetic of Social Experiences of Third Grade Children. J. Ed. Res. 30: 124-129; Oct. 1936.

Wallach, Lise and Sprott, Richard L. Inducing Number Conservation in Children. Child Develop. 35: 1057-1071; Dec. 1964.

Wallach, Lise; Wall, Jack A.; and Anderson, Lorna. Number Conservation: The Roles of Reversibility, Addition-Subtraction, and Misleading Perceptual Cues. Child Develop. 38: 425-442; Mar. 1967.

Wallen, Norman E. and Vowles, Robert O. The Effect of Intraclass Ability Grouping on Arithmetic Achievement in the Sixth Grade. J. Ed. Psychol. 51: 159-163; June 1960.

Walters, Margaret R. How to Study Arithmetic. Sch. Sci. Math. 34: 848-852; Nov. 1934.

Warden, C. J. and Cohen, A. A Study of Certain Incentives Applied Under Schoolroom Conditions. J. Genet. Psychol. 39: 320-327; 1931.

Washburne, Carleton W. Comparison of Two Methods of Teaching Pupils to Apply the Mechanics of Arithmetic to the Solution of Problems. El. Sch. J. 27: 758-767; June 1927.

Washburne, Carleton. When Should We Teach Arithmetic? - A Committee of Seven Investigation. El. Sch. J. 28: 659-665; May 1928.

Washburne, Carleton. Mental Age and the Arithmetic Curriculum: A Summary of the Committee of Seven Grade Placement Investigations to Date. *J. Ed. Res.* 23: 210-231; Mar. 1931.

Washburne, Carleton. The Values, Limitations, and Applications of the Findings of the Committee of Seven. *J. Ed. Res.* 29: 694-707; May 1936.

Washburne, Carleton. The Teaching of Linear Measure, Square Measure, and Time. *El. Sch. J.* 39: 509-523; Mar. 1939.

Washburne, Carleton W. and Morphett, Mabel Vogel. Unfamiliar Situations as a Difficulty in Solving Arithmetic Problems. *J. Ed. Res.* 18: 200-224; Oct. 1928.

Washburne, Carleton W. and Osborne, Raymond. Solving Arithmetic Problems. I. *El. Sch. J.* 27: 219-226; Nov. 1926.

Washburne, Carleton W. and Osborne, Raymond. Solving Arithmetic Problems. II. *El. Sch. J.* 27: 296-304; Dec. 1926.

Washburne, Carleton and Vogel, Mabel. Are Any Number Combinations Inherently Difficult? *J. Ed. Res.* 17: 235-255; Apr. 1928.

Weaver, J. Fred. A Crucial Problem in the Preparation of Elementary School Teachers. *El. Sch. J.* 56: 255-261; Feb. 1956.

Weaver, J. Fred. Patterns in Arithmetic: A Three-Year Report. *Arith. Teach.* 12: 291-293; Apr. 1965.

Weaver, J. Fred. Levels of Geometric Understanding: An Exploratory Investigation of Limited Scope. *Arith. Teach.* 13: 322-332; Apr. 1966.

Weaver, J. Fred. Levels of Geometric Understanding Among Pupils in Grades 4, 5, and 6. *Arith. Teach.* 13: 686-690; Dec. 1966.

Weaver, J. F. Nonmetric Geometry and the Mathematical Preparation of Elementary-School Teachers. *Am. Math. Mon.* 73: 1115-1121; Dec. 1966.

Welch, Livingston. The Development of Discrimination of Form and Area. *J. Psychol.* 7: 37-54; 1939.

Welch, Livingston. The Development of Size Discrimination Between the Ages of 12 and 40 Months. *J. Genet. Psychol.* 55: 243-268; 1939.

Welch, Ronald C. and Edwards, Charles W., Jr. A Test of Arithmetic Principles, Elementary Form. *Ind. U. Sch. Ed. Bull.* 41: 1-86; Sept. 1965.

Werner, Heinz. Perception of Spatial Relationship in Mentally Deficient Children. J. Genet. Psychol. 57: 93-100; 1940.

Werner, Heinz and Carrison, Doris. Measurement and Development of the Finger Schema in Mentally Retarded Children; Relation of Arithmetic Achievement to Performance on the Finger Schema Test. J. Ed. Psychol. 33: 252-264; Apr. 1942.

Wertheimer, J. E. Some Results of Monroe's Diagnostic Tests in Arithmetic. J. Ed. Psychol. 11: 109-112; Feb. 1920.

West, Jeff and Sievers, Callie. Experiment in Cross Grouping. J. Ed. Res. 54: 70-72; Oct. 1960.

Wetherington, Julia. Grade Placement of the Content of Arithmetic. El. Sch. J. 37: 41-46; Sept. 1936.

Wheat, H. Responses of School Children to Conventional Arithmetic Problems. Columbia Studies in Ed. 359: 1-124; 1928.

Wheat, Harry G. Research in Teaching Methods of Self-Instruction. Ed. Res. B. 20: 87-93, 112; Apr. 16, 1941.

Wheeler, Lester R. A Comparative Study of the Difficulty of the 100 Addition Combinations. Pedagog. Sem. and J. Gen. Psychol. 54: 295-312; June 1939.

Wheeler, Lester R. A Comparative Study of the Difficulty of Learning the Multiplication Combinations. Pedagog. Sem. and J. Gen. Psychol. 49: 189-206; Sept. 1941.

Wheeler, Lester R. and Wheeler, Viola D. An Experimental Study in Learning to Read Numerals. Math. Teach. 33: 25-31; Jan. 1940.

White, Helen M. Does Experience in the Situation Involved Affect the Solving of a Problem? Ed. 54: 451-435; Apr. 1934.

Whitman, Nancy C. In-Service Education and the Learning of Conceptual Mathematics. Arith. Teach. 13: 149-151; Feb. 1966.

Whitson, Willie E. Remedial Cases in Arithmetic, Case 4. Peabody J. Ed. 7: 362-372; May 1930.

Wilburn, D. Banks. A Method of Self-Instruction for Learning the Easier Addition and Subtraction Combinations in Grade I. El. Sch. J. 42: 371-380; Jan. 1942.

Wilburn, D. Banks. A Method of Self-Instruction for Learning the Thirty-Six Addition Combinations With Sums From Eleven to Eighteen and Their Corresponding Subtraction Facts in Grade II. Math. Teach. 38: 246-251; Oct. 1945.

Wilburn, D. Banks. Learning to Use a Ten in Subtraction. *El. Sch. J.* 47: 461-466; Apr. 1947.

Willey, Roy DeVerl. The Use of Arithmetic in the Out-of-School Life of Children. *Math. Teach.* 35: 23-28; Jan. 1942.

Willey, Roy DeVerl. Vocabulary for Arithmetic in the Elementary Grades. *El. Eng. R.* 19: 64-66; Feb. 1942.

Willey, Roy DeVerl. Arithmetical Processes Needed by Children. *El. Sch. J.* 42: 524-527; Mar. 1942.

Willey, Roy DeVerl. A Study of the Use of Arithmetic in the Elementary Schools of Santa Clara County, California. *J. Ed. Res.* 36: 353-365; Jan. 1943.

Williams, Alfred H. Mathematical Concepts, Skills and Abilities of Kindergarten Entrants. *Arith. Teach.* 12: 261-268; Apr. 1965.

Williams, Catharine M. Arithmetic Learning in an Experience Curriculum. *Ed. Res. B.* 28: 154-162, 167-168; Sept. 14, 1949.

Williams, Claude L. and Whitaker, Ruth L. Diagnosis of Arithmetic Difficulties. *El. Sch. J.* 37: 592-600; Apr. 1937.

Williams, Emmet D. and Shuff, Robert V. Comparative Study of S.M.S.G. and Traditional Mathematics Text Material. *Math. Teach.* 10: 495-504; Nov. 1963.

Williams, Mary Heard and McCreight, Russell W. Shall We Move the Question? *Arith. Teach.* 12: 418-421; Oct. 1965.

Wilson, Dorothy W. Teaching Denominate Numbers and Measures. *Ed. Meth.* 16: 177-181; Jan. 1937.

Wilson, Estaline. Improving the Ability to Read Arithmetic Problems. *El. Sch. J.* 22: 380-386; Jan. 1922.

Wilson, F. T. The Effect of the Form of a Combination in the Learning of a Multiplication Table by Bright and Dull Children. *J. Ed. Psychol.* 22: 536-537; Oct. 1931.

Wilson, Guy M. Promise and Performance in Arithmetic Drill Service. *J. Ed. Res.* 19: 169-176; Mar. 1929.

Wilson, Guy M. New Standards in Arithmetic: A Controlled Experiment in Supervision. *J. Ed. Res.* 22: 351-360; Dec. 1930.

Wilson, Guy M. For 100 Percent Subtraction, What Method? A New Approach. J. Ed. Res. 27: 503-507; Mar. 1934.

Wilson, Guy M. A New Incidence of Learning for the Fundamentals of Arithmetic. J. Ed. Res. 33: 425-433; Feb. 1940.

Wilson, Guy M. The Social Utility Theory as Applied to Arithmetic, Its Research Basis, and Some of Its Implications. J. Ed. Res. 41: 321-337; Jan. 1948.

Wilson, Guy M. Toward Perfect Scores in Arithmetic Fundamentals. Arith. Teach. 1: 13-17; Dec. 1954.

Wilson, Guy M. and Cassell, Mabel. A Research on Weights and Measures. J. Ed. Res. 46: 575-585; Apr. 1953.

Wilson, Guy M. and Dalrymple, Charles O. Useful Fractions. J. Ed. Res. 30: 341-347; Jan. 1937.

Wilson, Guy M. and Hanley, Gertrude L. For Basic Drill in Arithmetic, What Norm or Average is Satisfactory? Math. Teach. 32: 175-178; Apr. 1939.

Wilson, Guy M. and Porter, Everett. In the Useful Fundamentals of Arithmetic is it Possible to Overcome Temporary Lapses in Ability Leading to Steady Increase in Error? J. Exp. Ed. 5: 71-74; Sept. 1936.

Wilson, John A. R. Differences in Achievement Attributable to Different Educational Environments. J. Ed. Res. 52: 83-93; Nov. 1958.

Wilson, J. A. R. Achievement, Intelligence, Age, and Promotion Characteristics of Students Scoring at or Below the Tenth Percentile on the California Test of Personality. J. Ed. Res. 52: 283-292; Apr. 1959.

Wilson, John W. The Role of Structure in Verbal Problem Solving. Arith. Teach. 14: 486-497; Oct. 1967.

Wilson, W. K. and Ashbaugh, E. J. Achievement in Rural and Consolidated Schools. Ed. Res. B. 8: 358-363; Nov. 6, 1929.

Winch, W. H. Accuracy in School Children: Does Improvement in Numerical Accuracy Transfer? J. Ed. Psychol. 1: 557-589; Dec. 1910.

Winch, W. H. Further Work on Numerical Accuracy in School Children: Does Improvement in Numerical Accuracy Transfer? J. Ed. Psychol. 2: 262-271; May 1911.

Winer, Gerald A. Induced Set and Acquisition of Number Conservation. Child Develop. 39: 195-205; Mar. 1968.

Wirszup, Izaak. Current School Mathematics Curricula in the Soviet Union and Other Communist Countries. Math. Teach. 52: 334-346; May 1959.

Wittich, Walter A. A Number Readiness Test. Sch. Exec. 61: 11-13; Mar. 1942.

Wittrock, M. C. and Keislar, Evan R. Verbal Cues in the Transfer of Concepts. J. Ed. Psychol. 56: 16-21; Feb. 1965.

Wohlwill, Joachim F. Absolute vs. Relational Discrimination on the Dimension of Number. J. Genet. Psychol. 96: 353-363; 1960.

Wohlwill, Joachim F. A Study of the Development of the Number Concept by Scalogram Analysis. J. Genet. Psychol. 97: 345-377; Dec. 1960.

Wohlwill, Joachim. The Learning of Absolute and Relational Number Discrimination by Children. J. Genet. Psychol. 101: 217-228; July 1963.

Wohlwill, Joachim F. and Lowe, Roland C. Experimental Analysis of the Development of the Conservation of Number. Child Develop. 33: 153-167; 1962.

Wolf, William C. Non-Written Figuring: Its Role in the Elementary-School Curriculum. Ed. Res. B. 39: 206-213; Nov. 9, 1960.

Woodrow, Herbert. Intelligence and Improvement in School Subjects. J. Ed. Psychol. 36: 155-166; Mar. 1945.

Woody, Clifford. The Advantage of Ability Grouping. Ind. U. Sch. Ed. Bull. 1: 38-60; Jan. 1925.

Woody, Clifford. Results From Successive Repetitions of Certain Arithmetic Tests. Ind. U. Sch. Ed. Bull. 1: 61-79; Jan. 1925.

Woody, Clifford. Knowledge of Arithmetic Possessed by Young Children. Ind. U. Sch. Ed. Bull. 6: 50-85; Apr. 1930.

Woody, Clifford. Some Investigations Resulting From the Testing Program in Arithmetic: An Investigation to Determine the Influence of Specialized Drill in Reading Upon the Solution of Verbal Problems (1st of 3 studies). Ind. U. Sch. Ed. Bull. 6: 30-39; Apr. 1930.

Woody, Clifford. Some Investigations Resulting From the Testing Program in Arithmetic: An Investigation to Determine the Transfer Effects of Three Different Methods of Teaching Three Different Types of Examples in Two-Place Addition (2nd of 3 studies). Ind. U. Sch. Ed. Bull. 6: 39-45; Apr. 1930.

Woody, Clifford. Some Investigations Resulting From the Testing Program in Arithmetic: Case Study of a Girl in Grade VIIB Who Was Doing Unsatisfactory Work in Arithmetic (3rd of 3 studies). *Ind. U. Sch. Ed. Bull.* 6: 45-49; Apr. 1930.

Woody, Clifford. Achievement in Counting by Children in the Primary Grades. *Childhd. Ed.* 7: 339-345; Mar. 1931.

Woody, Clifford. The Arithmetical Backgrounds of Young Children. *J. Ed. Res.* 24: 188-201; Oct. 1931.

Woody, Clifford. Nature and Amount of Arithmetic in Types of Reading Material for the Elementary Schools. *Ed. Outlook* 6: 199-217; May 1932.

Worthen, Blaine R. Discovery and Expository Task Presentation in Elementary Mathematics. *J. Ed. Psychol.* 59: 1-13; Feb. 1968.

Worthen, Blaine R. A Study of Discovery and Expository Presentation: Implications for Teaching. *J. Teach. Ed.* 19: 223-242; Summer 1968. (see Worthen, Blaine R. Discovery and Expository Task Presentation in Elementary Mathematics. *J. Ed. Psychol.* 59: 1-13; Feb. 1968.)

Wozencraft, Marian. Sex Comparisons of Certain Abilities. *J. Ed. Res.* 57: 21-27; Sept. 1963.

Wozencraft, Marian. Are Boys Better Than Girls in Arithmetic? *Arith. Teach.* 10: 486-490; Dec. 1963.

Wrightstone, J. Wayne. An Evaluation of the Integrated Curriculum in the Upper Grades. *El. Sch. J.* 35: 583-587; Apr. 1935.

Wrightstone, J. Wayne. Achieving Skills in Progressive Primary Schools. *Ed. Meth.* 15: 35-38; Nov. 1935.

Wrightstone, J. Wayne. Evaluation of the Experiment With the Activity Program in the New York City Elementary Schools. *J. Ed. Res.* 38: 252-257; Dec. 1944.

Wrigley, Jack. The Factorial Nature of Ability in Elementary Mathematics. *Br. J. Ed. Psychol.* 28: 61-78; Feb. 1958.

Yeager, John L. and Lindvall, C. M. An Exploratory Investigation of Selected Measures of Rate of Learning. *J. Exp. Ed.* 36: 78-81; Winter 1967.

Young, Robert V. and McIsaac, John S. The Sequence of Processes Affects the Pupil's Interpretation of Verbal Problems in Arithmetic. *Ed.* 61: 488-491; Apr. 1941.

Zahn, Karl G. Use of Class Time in Eighth-Grade Arithmetic. Arith. Teach. 13: 113-120; Feb. 1966.

Zaslow, Robert W. Reversals in Children as a Function of Body Orientation. J. Ed. Psychol. 57: 133-139; June 1966.

Zeiler, Michael D. and Gardner, Ann M. Intermediate Size Discrimination in Seven- and Eight-Year Old Children. J. Exp. Psychol. 71: 203-207; Feb. 1966.

Zweng, Marilyn J. Division Problems and the Concept of Rate. Arith. Teach. 11: 547-556; Dec. 1964.

Zyve, Claire. An Experimental Study of the Teaching of Arithmetic Combinations. Ed. Meth. 12: 16-18; Oct. 1932.

Zyve, Claire and Smith, Evelyn. The Effect of Radio Musical Accompaniment on Accomplishment in School Work. Ed. Meth. 13: 366-368; Apr. 1934.

- - -. Time for Mathematics! Br. El. Math J. 1: 56-57; Autumn 1963.